Painlevé and the general relativity

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Abstract

The study of the Painlevé papers, published in 1921-1922, analyzing differences between the Newtonian theory and the general relativity, reveals an incredible creativity even beyond the still inspired oriented formalism described in his first paper which was the first in the history erasing the singularity on the horizon allowing it to be inwards crossed over by an observer. This spacetime incredible character baffled even the most brilliant minds and the subsequent debate at the "Académie des Sciences" whether it produced masterful contributions, sank into oblivion.

Painlevé proposed also a space covariant geometrical formalism for deriving the equations of geodesic motion in the Newtonian theory. This does not longer involve the absolute time of the classical mechanics but takes the proper time, affine parameter of this geodesic, as dynamical parameter. Whether the extension of this space formalism to general relativity will not fully succeed, it still will yield a correct result for describing the conformal spacetime structure of the Schwarzschild problem in general relativity. One might wonder how, a scientist, not familiar with the general relativity, as Painlevé was able to set up such breaking concepts of time and space in his work. Brought to light with honors in recent papers, we will show how, by using various formal descriptions suitable for this spacetime, the Painlevé formalism reveals better than any other the underlying physics and its epistemological involvements. The controversial Painlevé 's claims about the ds^2 did not deserve to be struck by such ostracism. At the end, we will stress convergence of historical, epistemological and scientific approaches.

Historical overview: Painlevé-Gullstrand (PG) coordinates were proposed independently by Paul Painlevé in 1921 [1] and Allvar Gullstrand [2] in 1922. Aimed to describe the same spacetime solution as the Schwarzschild coordinates, in 1921, October 24, in a first paper, among some doubts and critics on GR, Painlevé proposed a GR line element using coordinates which were the first ones in history to be non-singular at the horizon, allowing one to describe timelike or null worldlines inward crossing the horizon. As Schwarzschild coordinates have a coordinate singularity at the event horizon, at that time the horizon was considered either as an artifact of the theory or as the ultimate border of spacetime. The "trick" of the Painlevé proposal was that he no longer stuck to a full quadratic (static) form but instead, allowed a cross time-space product making the metric form no longer static but stationary and no longer symmetric but oriented. The orientation of the metric form was necessary, in these type of coordinates, for allowing crossing the horizon inward but not outward (oriented phenomenology).

In a second, longer paper (november 14th 1921),[3] Painlevé explains how he derived his solution: not from the Schwarzschild solution, but directly from a generic spherically symmetric form, constrained by the Einstein equation. The result was quite a smart form depending on only two functions of the r coordinate satisfying identically the Einstein equation, for any function, yielding a double infinity of solutions.

Painlevé wrote to Einstein for introducing his solution and invited Einstein to Paris for a debate. In Einstein reply letter (december 7th) Einstein apologized for not being in a position to come soon, and explained why he was not pleased with Painlevé's arguments, critics and coordinates. Finally,

1 2013, january 9th : original language : french. (639 pages). Email : jacques.fric@neuf.fr

[1] Paul Painlevé, "La mécanique classique et la théorie de la relativité", C. R. Acad. Sci. (Paris) 173, 677–680(1921).

[2] Allvar Gullstrand, "Allgemeine Lösung des statischen Einkörperproblems in der Einsteinschen Gravitationstheorie", Arkiv. Mat. Astron. Fys. 16(8), 1–15 (1922).

[3] "La gravitation dans la mécanique de Newton et dans la mécanique d'Einstein" C.R Acad. Sci. (Paris) 173, 873-886(1921).

Einstein came to Paris in early April. On the 5th of April 1922, in a debate at the "Collège de France" with Painlevé, Becquerel, Brillouin, Cartan, De Donder, Hadamard, Langevin and Nordmann on "the infinite potentials", Einstein, baffled by the non quadratic term in the line element, rejected the Painlevé solution. That signed the death (for a long time) of the Painlevé proposal.