Summary

An examination of the contributions of Painlevé, reveals an incredible wealth of creativity that go far beyond the innovative form he proposed in 1921. But the ensuing debate, in the Academy of Sciences, as the analysis will show, even if it produced masterful contributions, will sink into oblivion without saving essentials.

Painlevé himself, despite a commendable attempt at clarification, under pressure from the scientific community, has too quickly abandoned.

We analyze how the fondamental concepts of space and time , as used in this innovative solution, have put into trouble even the most eminent contemporary scientists, including Einstein. Therefore, all of them rejected his proposal, wrongly.

We discuss the conditions for the emergence of such new theories and solutions breaking with current ontological concepts.

The context: Einstein

When Einstein, professor at Berlin, published his final equation of general relativity, at the end of November 1915, the World War I was raging. Nevertheless he will be comforted in his theory by:

- Exact prediction of the advance of the perihelion of Mercury (1915).

- Prediction of light deflection by the Sun confirmed (1919).

First exact solution in 1916 (Schwarzschild) for the single spherically symmetrical body (solar system). But these equations are singular on a surface at a certain distance from the center: Physical meaning?

This is a problem for Einstein and the few supporters (at that time) of his theory. Relativists try to conceal the "monstrosity" (sic Eddington) by considering it as a mathematical artifact.

The context: Painlevé

At that time Painlevé, who will be minister of War (1917) and, later in the same year, head of the french government in 1917 (for 3 months only), is invested in the war effort.

After the end of the war, he will stimulate the debate that begins at the Academy of Sciences by issuing a critically but constructive proposal in 1921.

Meanwhile, Einstein was awarded the Nobel Prize for "his contributions to physics, especially for quantum theory", but not for the general relativity, still controversial, even though the book of the famous mathematician H. Weyl, "Raum, Zeit, Materie", which devoted a large part at the Einstein's theory contributes to increase the credibility in the general relativity.

ACADÉMIE DES SCIENCES.

SÉANCE DU LUNDI 24 OCTOBRE 1921.

PRÉSIDENCE DE M. GEORGES LEMOINE.

MÉMOIRES ET COMMUNICATIONS

DES MEMBRES ET DES CORRESPONDANTS DE L'ACADÉMIE.

MÉCANIQUE. — *La Mécanique classique et la théorie de la relativité.* Note de M. PAUL PAINLEVÉ.

axes. Cette hypothèse admise, les einsteiniens parviennent au ds^2 (à quatre variables) aujourd'hui célèbre, dont les géodésiques définissent dans leur théorie le mouvement du point gravitant, à savoir

$$(1) \qquad ds^2 = dt^2\left(1 - \frac{a}{r}\right) - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) - \frac{dr^2}{1 - \frac{a}{r}},$$

a désignant une constante arbitraire que déterminera la masse du centre matériel O.

Mais ce ds^2 n'est pas le seul qui réponde à toutes les conditions einsteiniennes. Il en est une infinité d'autres dépendant de deux fonctions de r et le choix de la formule (1) entre toutes ces formules est purement arbitraire. Parmi ces formules il en est d'aussi simples que la formule (1) et qui entraînent exactement les mêmes vérifications. Telle celle-ci :

$$(2) \qquad ds^2 = dt^2\left(1 - \frac{a}{r}\right) + 2\, dr\, dt\sqrt{\frac{a}{r}} - d\sigma^2,$$

avec

$$d\sigma^2 = dr^2 + r^2[d\theta^2 + \sin^2\theta\, d\varphi^2]$$

The embrace of space and time in spacetime, revealed by the non-quadratic term dr.dt.

The Schwarzschild metric, where all terms are quadratic for time or space, is singular on the horizon.

The Painlevé form is the first one in history to be non singular on the horizon, but as it includes a non quadratic term that mixes space and time, this has baffled the scientific community !

What would be the physical meaning of a term that combines space and time which are physically different into a metric, and in addition implies an orientation of the metric.

This solution, not singular on horizon reveals that it is not impassable, unlike the suggestion of Schwarzschild solution . One can pass through it, inwards but not outwards, so with no return possibility.

The spatio-temporal orientation is the key for understanding the physical structure of the solution.

Beyond this horizon, space and time exhibit strange properties. Physical time can exist only when associated to movement.

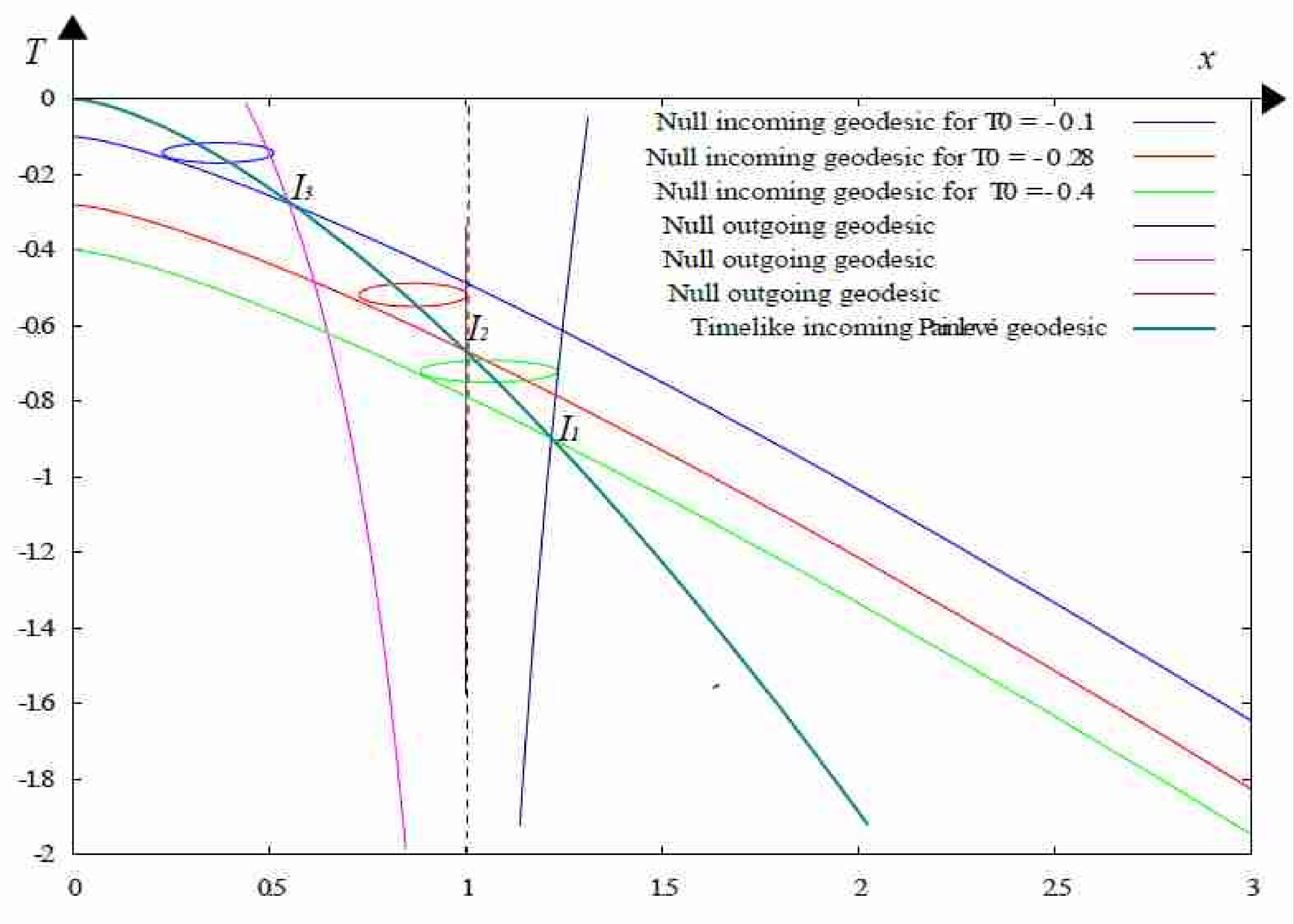
The orientation of space implies an asymmetry in the structure of space-time.

It induces, in addition, two symmetrical regions, involving a dissociation of vacuum in two spacetimes of opposite orientations. Although Painlevé's statement did not report it, this was obvious!

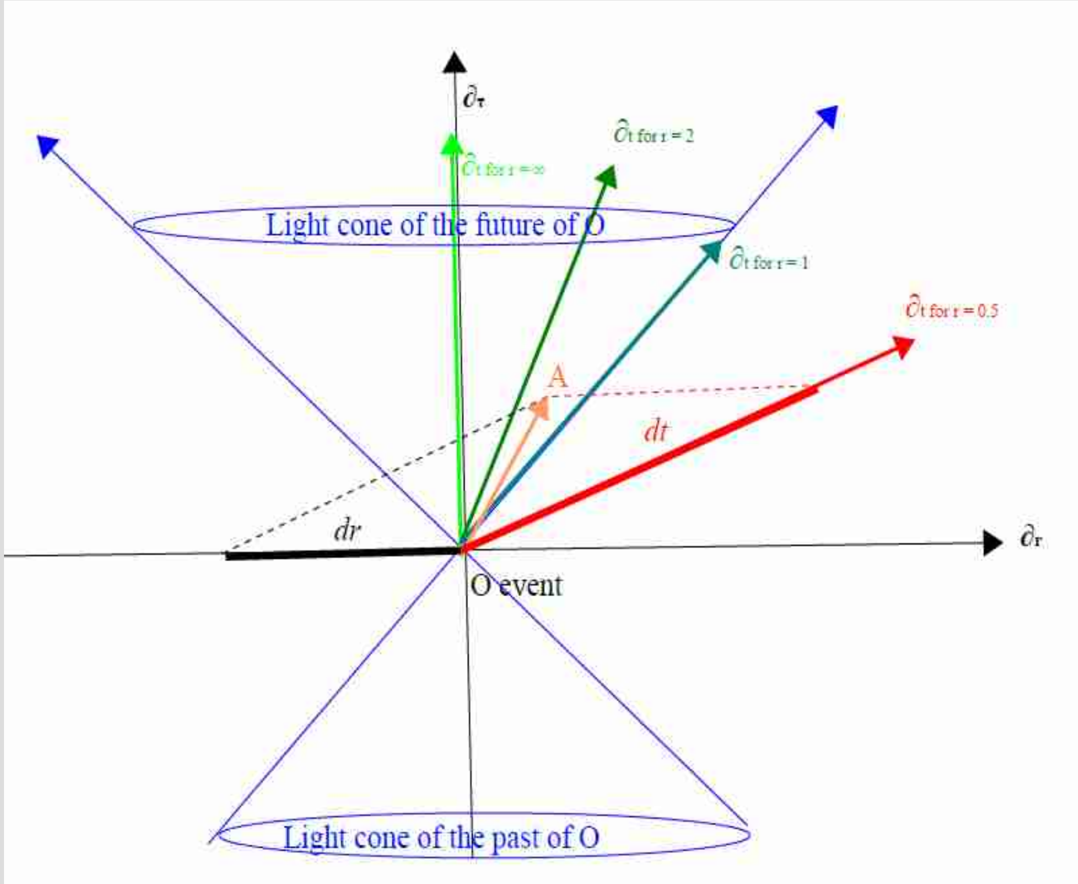
This innovative proposal was not understood, especially the orientation of the solution, key concept necessary for crossing the horizon, (oriented phenomenology). It is that character with its implications which unifies the two sub-regions of the Schwarzschild form.

But, possibly overwhelmed by the scope of his discovery, Painlevé, renounces (temporarily) to it, wrongly, 3 weeks later!

An effect of orientation: The tilting of light cones of the local observer in Painlevé's form.



Dual representation of the previous one: The local observer sees a symmetrical space-time .



Now we look at the spacetime through the local light cone (constant) of the observer.
We set $2GM/c^2 = 1$.
The vectors ∂_t and ∂_r locally tangent to the coordinates lines t and r of the Painlevé metric form are represented for four values of r.
For $r > 1$, the vector ∂_t is inside the light cone. Static timelike worldlines are possible.
For $r < 1$ the vector ∂_t (in red) is outside the light cone. Static timelike worldlines are not possible, but for dr negative, the resulting vector (OA) may be inside the light cone, involving a timelike worldline.

This illustrates the distrust of scientists towards an oriented spacetime. Hence the "principle of reversibility" Painlevé! The apparent contradiction between the two representations is the manifestation of the curvature of spacetime!

How the form of Painlevé shows that the horizon paradox in the form of Schwarzschild is fictional!

Chart in Schwarzschild's coordinates, *(t, r)*

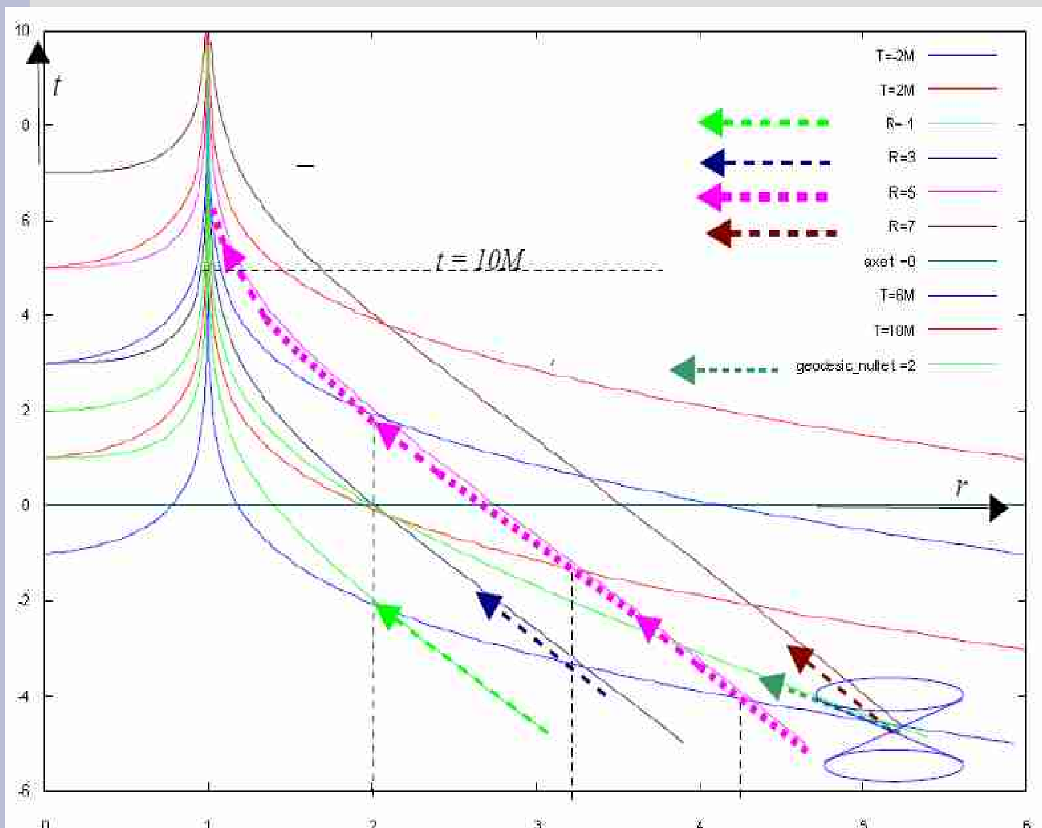
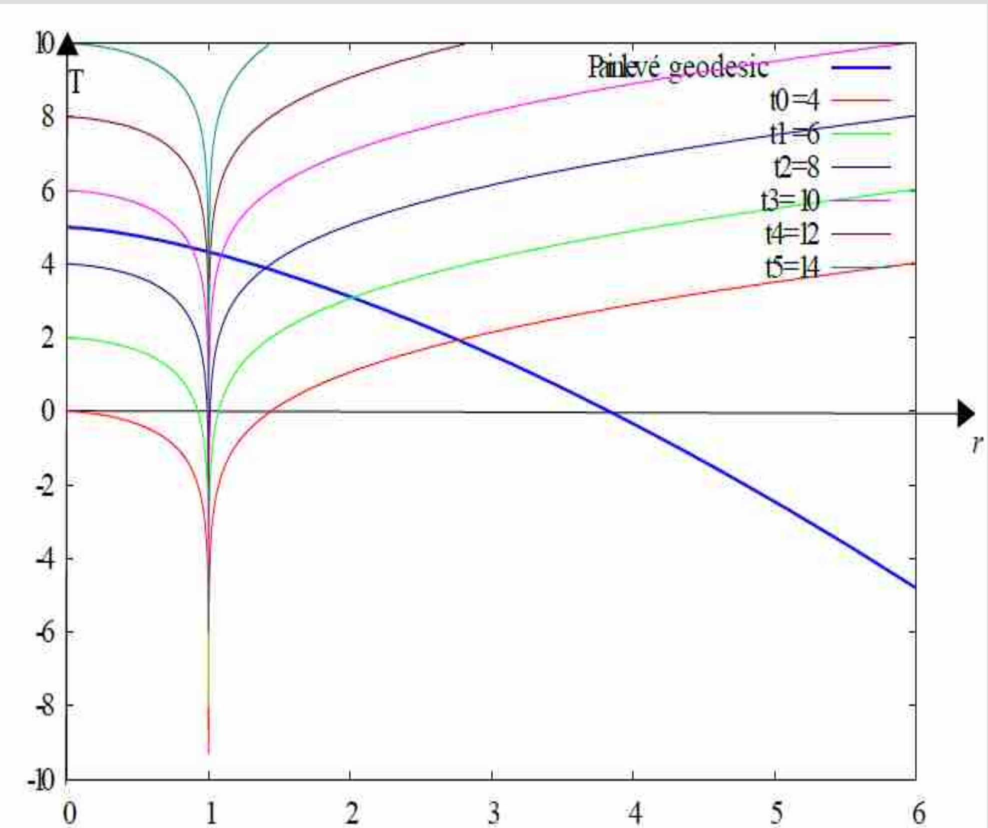


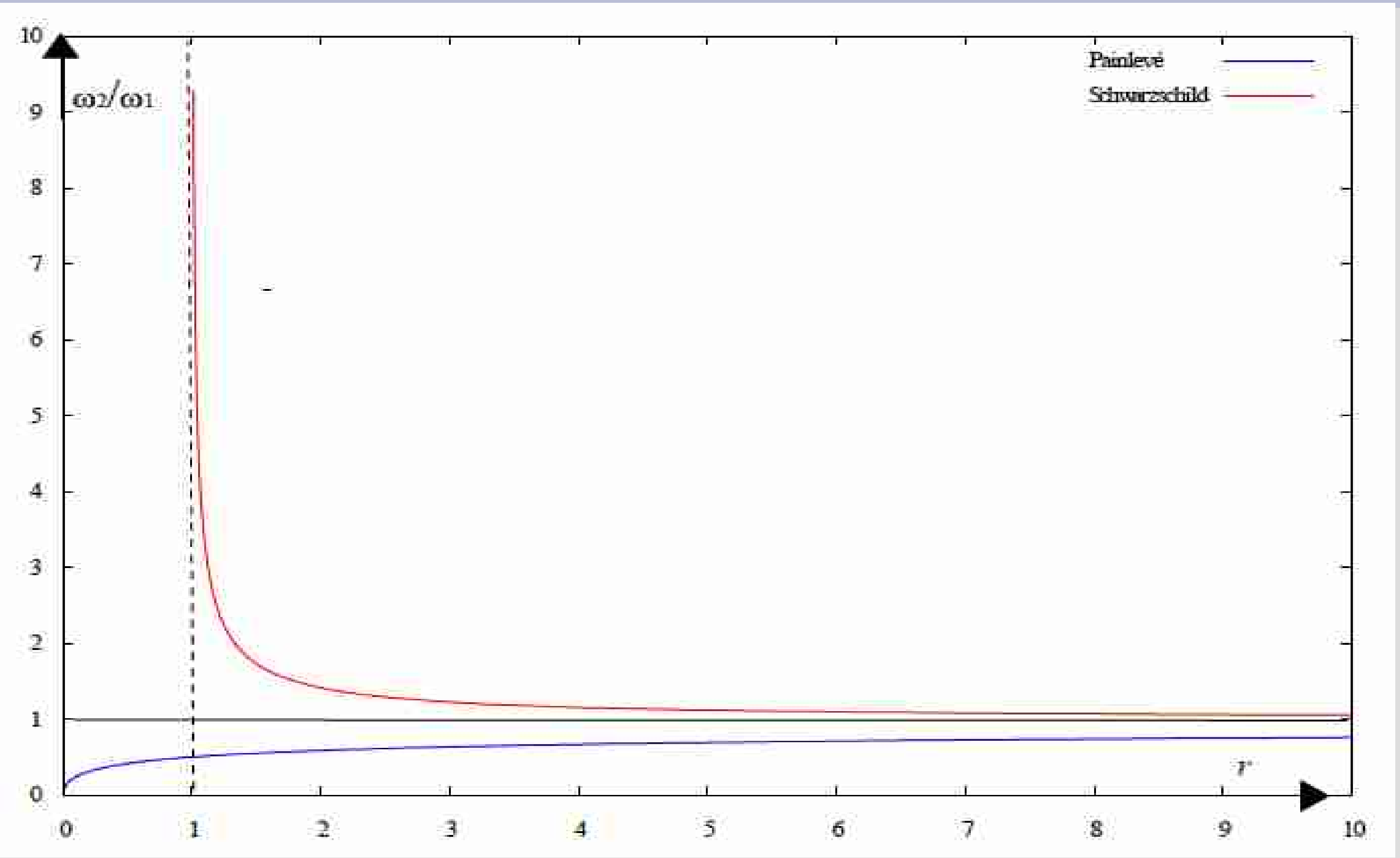
Chart in Painlevé's coordinates, *(T,r)*



Four Painlevé's geodesics R = i (bold dashed arrows), one null geodesic, one light cone and four isochronous (in Painlevé's coordinate) lines, T = j, are represented. As all lines reach infinity for r=1, it is difficult to conclude anything in this diagram.

One Painlevé's geodesic (blue), six isochronous lines (Schwarzschild's temporal coordinate). Painlevé's geodesic crosses all isochronous lines of Schwarzschild's temporal coordinate before crossing the horizon in a finite proper time.

Redshift of incoming light: Painlevé vs Schwarzschild



Light is supposed to come from infinity with a frequency ω_1.
The chart gives the ratio ω_2/ω_1, where ω_2 is the frequency measured either by a static observer or a by Painlevé's observer, at a finite distance r.
Notice the opposite phenomenology.

ACADÉMIE DES SCIENCES.

SÉANCE DU LUNDI 14 NOVEMBRE 1921.

PRÉSIDENCE DE M. GEORGES LEMOINE.

MÉMOIRES ET COMMUNICATIONS

DES MEMBRES ET DES CORRESPONDANTS DE L'ACADÉMIE.

MÉCANIQUE. — *La gravitation dans la Mécanique de Newton et dans la Mécanique d'Einstein.* Note de M. PAUL PAINLEVÉ.

Il suit de là, comme on voit, qu'on peut donner à la théorie de la gravitation newtonienne la forme suivante (principe de la moindre action) : *Les trajectoires du point* P *sont les géodésiques du* ds^2

$$ds^2 = (U + h)(dx^2 + dy^2 + dz^2), \qquad (h \text{ constante arbitraire}),$$

où U *est une fonction de* x, y, z *qui s'annule à l'infini dont le* ΔU *est nul à l'extérieur de la sphère* S *et est égal à une constante négative dans* S.

Unusual but enlightening properties, emerging from Painlevé's equation

To compare the two theories in this solution, Painlevé derives a covariant geometric formalism, as in general relativity, for the Newtonian mechanics, but only spatial, without any references to the Newtonian (absolute) time.

In classical mechanics, the length of a plane curve, defined by $r = f(\varphi)$ in polar coordinates, is computed by using the Euclidean metric. The equation of motion on this curve is given elsewhere.

We can also consider that this length is the affine parameter (λ) of this curve. In this case, the curve is defined by two functions: $r(\lambda)$ and $\varphi(\lambda)$. As λ is not used in the equation $r = f(\varphi)$, without altering the relation, which defines the curve, we have the freedom to apply a gauge transformation on this affine parameter.

This allows, considering this affine parameter as the dynamic geodesic motion parameter, to unify the formalism.

The geometric formalism of Newtonian mechanics to derive the laws of motion give rise to a physical (proper) time which marked the end of absolute time

This is the meaning of the proposal of Painlevé who, in fact, defines an affine parameter, by using the *gauge freedom for* taking into account the action of a field, a process in full compliance with Weyl's ideas about the gauge theory, at that time.

Moreover, the time, as dynamic parameter, will emerge naturally via the geodesic equation depending only on the spatial geometry, the curvature of which is determined solely by the gravitation. This confers it a natively physical status and seals its relationship with the space and the physics.

One can check that this concept of time, derived from λ, affine parameter of the spatial geodesic is equivalent to the Newtonian absolute time t, by setting $t = i\lambda$. This yields a $(-dt^2 + d\sigma^2)$ similar to the relativistic form.

Therefore, Newtonian's time can be ignored in these solutions. This allows to replace the hybrid formalism (2 equations) by an homogeneous formalism (1 equation) and reveals the relational nature of the time.

The difference in nature of space and time are formally specified

The imaginary number *i,* introduced in these equations, characterizes the physical difference between time and space such as specified in the unified geometric formulation of general relativity.

But, unlike relativity where the “formally imaginary” time is a coordinate involved in a geometrical invariant (ds^2) which in turn can be physically timelike, here we get directly a “formally imaginary” physical proper time defined by a physical constraint (energy conservation).

This results from the identity of the geodesics defined by the geometrical form of Painlevé and by the classical Newtonian formalism, this implies that the “imaginary” character of time is also existing in the Newtonian theory, even though it is not explicit, the equations for time and space being separated.

But, when we attempt to unify the formalism such as in the Painlevé proposal (eq. 2-4), where time and space are parts of a unified equation, the difference in nature is exhibited by the imaginary number *i* attached to the time.

Newtonian time and proper time identified as the same dynamic parameter

As the absolute Newtonian time t is not a coordinate such as the spacelike coordinates of a geometric form, the Newtonian gravitation is not a spacetime theory requiring a timelike coordinate along with spacelike coordinates.

The Newtonian time is conceptually a dynamic physical parameter.

This will make ontologically possible its equivalence with the relativistic proper time, affine and dynamic parameter of a timelike worldline.

This is an epistemological convergence with the relativistic approach, where the proper time as measured by the observer, affine parameter of the worldline of this observer, is the dynamic parameter of this physical system.

In this formulation, the time will lose its *a priori* "metaphysical" character.

Emergence of a physical time : Summary

According to the spatial approach of Painlevé, we see that the absolute Newtonian time is absent.

The affine parameter, dynamic parameter, is the proper time of an observer in geodesic motion. This geodesic is defined in a conformal Euclidean spatial geometry, the conformal factor (scalar potential in Newtonian gravitation) being determined by the gravitational field.

This geometry determines the geodesic, therefore its affine parameter. As the curvature of this geometry depends only on the gravitational field, we are entitled to say that the proper time that we consider, emerges from physics, namely, from the geodesic motion of a body submitted to gravity in our case.

So, unlike Newtonian mechanics where space and time were *a priori* concepts and where the motion is a “byproduct”, here time emerges from motion, *i.e.* from the geodesic resulting from gravity!

Painlevé explicits in a 2nd article (11/14/1921), how he derived the line element of the metric of his 1st article.

Starting from the generic form: $ds^2 = A(r)dt^2 - 2B(r)dt\,dr - C(r)[r^2(d\theta^2 + \sin^2\theta\, d\varphi)] - D(r)dr^2$.[95] ([3])

He continues:

IV. *Conditions einsteiniennes invariantes.* — Quelles que soient d'ailleurs les fonctions A, B, C, D de r que l'expérience nous conduirait à adopter, il serait toujours possible de former des conditions invariantes auxquelles devraient satisfaire les coefficients de ds^2 quand on y remplace r, θ, φ et t en fonction de quatre variables entièrement quelconques. Mais Einstein veut *a priori* que ces conditions invariantes soient des équations aux dérivées partielles du deuxième ordre d'une forme spéciale, qui s'inspirent à la fois des théories de la gravité newtonienne en coordonnées curvilignes, et de la théorie de la courbure des surfaces ordinaires.

Ce sont ces restrictions capitales, et non le truisme pur et simple de l'invariance, qui parmi les ds^2 de la forme (3) ne laissent subsister que les suivants :

$$(4)\quad ds^2 = \left[1 - \frac{2\mu}{f(r)}\right][dt - \chi(r)\,dr]^2 - f^2(r)[d\theta^2 + \sin^2\theta\, d\varphi^2] - \frac{f'^2(r)\,dr^2}{1 - \frac{2\mu}{f(r)}},$$

où μ est une constante et où f et χ sont deux fonctions arbitraires de r telles seulement que $\chi(r)$ tende vers zéro et $f'(r)$ (toujours positif) tende vers 1, quand r tend vers l'infini.

Equation (4) is remarkable, it satisfies the Einstein equation (in vacuum) whatever the functions *f(r)* and *χ(r)*, thus defining a doubly infinite class of solutions. For $f(r) = r$ with $\chi(r) = 0$, one got Schwarzschild's form and with $\chi(r) = (2M/r)^{1/2}/(1-2M/r)$, Painlevé's form.

Then, he renounces invoking constraints on the properties of space

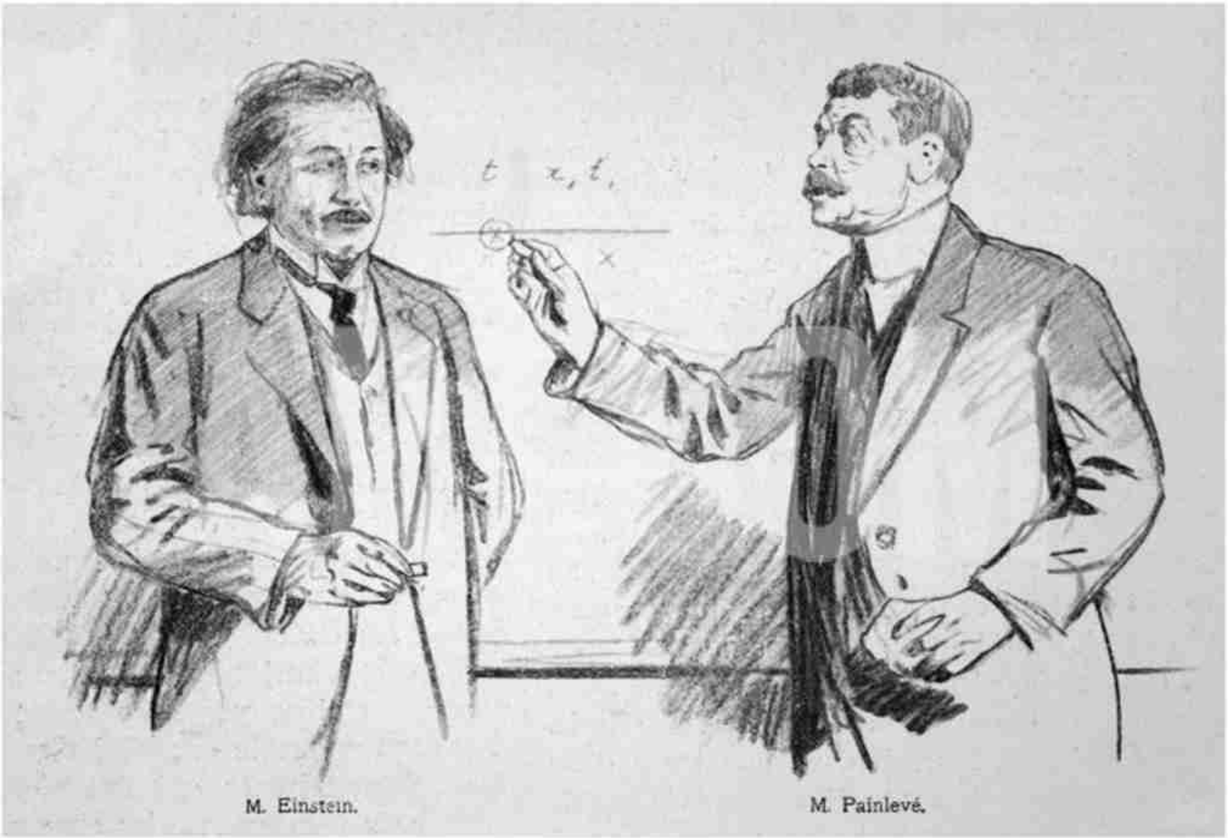
Painlevé invokes a principle of reversibility of space for invalidating the solutions with non-quadratic terms. He surrenders, wrongly, to the criticism and misunderstanding of his contemporaries, raised by his proposal.

The trouble comes from using pseudo-Riemannian's geometry in general relativity. This led the mathematicians, as evidenced by the work of H. Weyl on "problem of space" and those of E. Cartan, to formalize, at local infinitesimal scale, the concept of space in this new framework.

One problem was to reconcile the local infinitesimal homogeneous character of the space with its global character which was not. In 1921, this work was still in progress and we understand that the scientists lacked references to tackle these issues.

But later, during the visit of Einstein in Paris (April 7th 1922), he will defend fiercely his proposal, alone against all, but Einstein's disapproving statement will sign the death warrant for it, for a long time.

The meeting of April 5, 1922 in Paris



A polite but fierce debate : Sketch published in « L'illustration 04/22»

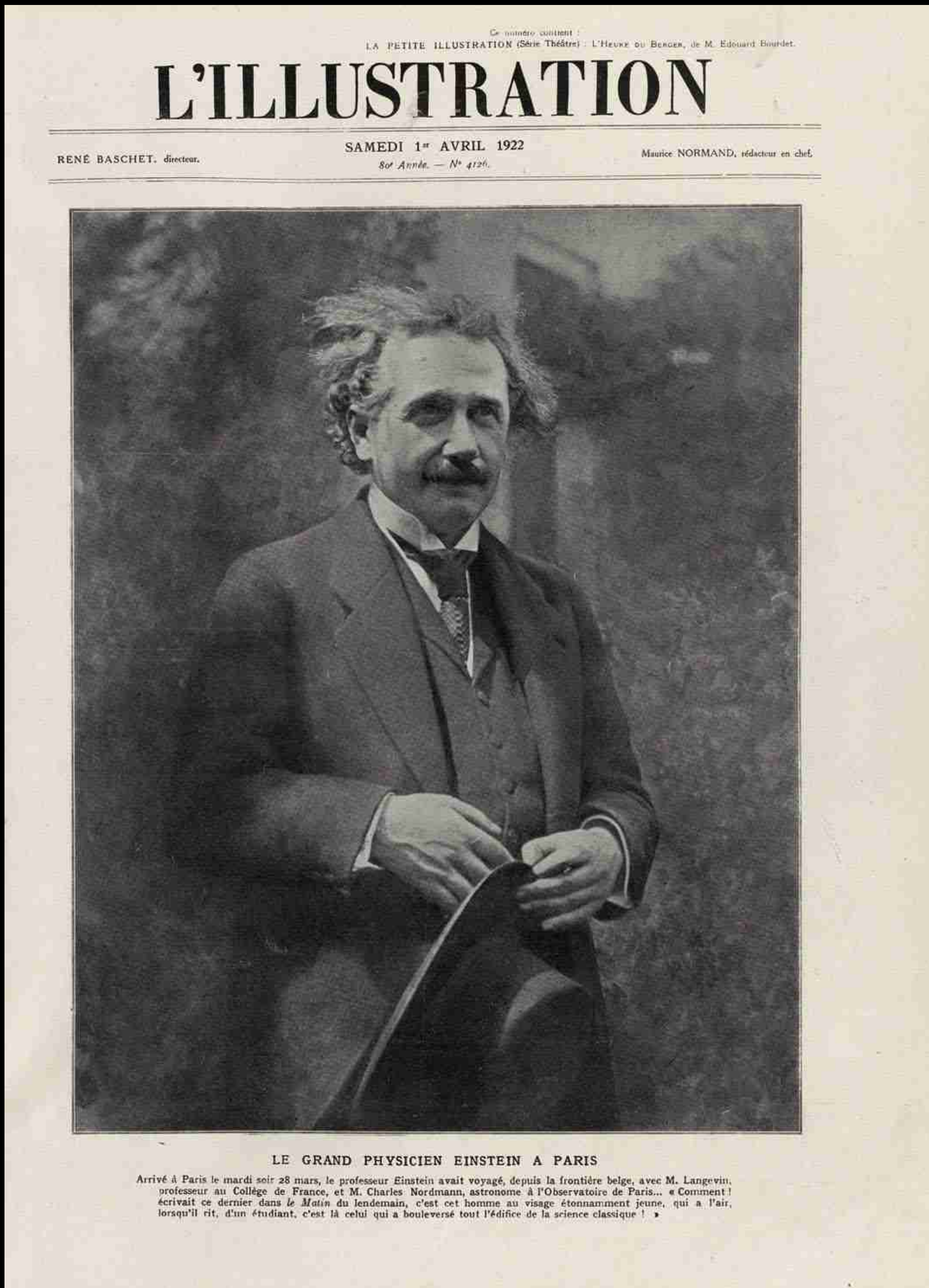
Langevin & Einstein (April 1922)

Painlevé 1923

The debate on possible solutions of the Einstein equations for the Schwarzschild problem was highlighted during Einstein's visit at the « Collège de France » on April 7th 1922. Painlevé, helped by his colleague, J. Hadamard, opened the debate on the Schwarzschild singularity (horizon), called by the attendance « the Hadamard catastrophe ». Einstein, rejected (wrongly) definitly the Painlevé proposal as a possible solution. It will take a long time, for repairing this mistake :Reported by Charles Nordmann, in « Einstein in Paris », http://www.21stcenturysciencetech.com/Articles_2011/Summer-2011/Einstein_Paris.pdf

1922 Einstein at the Collège de France invited by Langevin

COLLÈGE DE FRANCE 1530



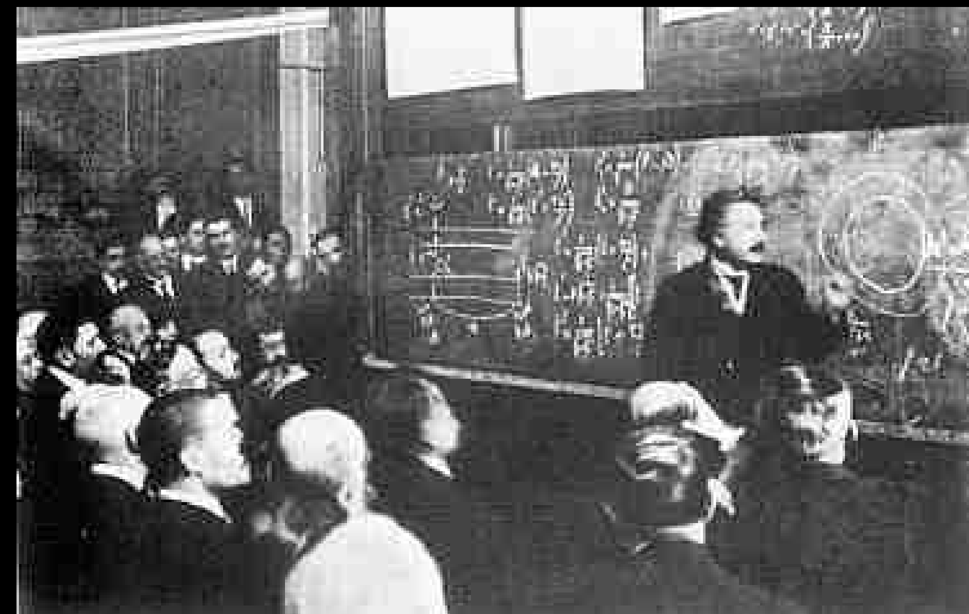
Ce numéro contient :
LA PETITE ILLUSTRATION (Série Théâtre) : L'HEURE DU BERGER, de M. Edouard Bourdet.

L'ILLUSTRATION

RENÉ BASCHET, directeur. SAMEDI 1er AVRIL 1922 — 80e Année. — N° 4126. Maurice NORMAND, rédacteur en chef.

LE GRAND PHYSICIEN EINSTEIN A PARIS

Arrivé à Paris le mardi soir 28 mars, le professeur *Einstein* avait voyagé, depuis la frontière belge, avec M. Langevin, professeur au Collège de France, et M. Charles Nordmann, astronome à l'Observatoire de Paris... « Comment ! écrivait ce dernier dans *le Matin* du lendemain, c'est cet homme au visage étonnamment jeune, qui a l'air, lorsqu'il rit, d'un étudiant, c'est là celui qui a bouleversé tout l'édifice de la science classique ! »



At the « maison des polytechniciens »

ACADÉMIE DES SCIENCES.

SÉANCE DU LUNDI 1er MAI 1922.

PRÉSIDENCE DE M. ÉMILE BERTIN.

MÉMOIRES ET COMMUNICATIONS

DES MEMBRES ET DES CORRESPONDANTS DE L'ACADÉMIE

MÉCANIQUE. — *La théorie classique et la théorie einsteinienne de la gravitation.* Note (¹) de M. PAUL PAINLEVÉ.

Le ds^2 est alors nécessairement de la forme

$$(6) \qquad ds^2 = \frac{dt^2}{U(x_1, x_2, x_3)} - d\sigma^2 \qquad (U > 0),$$

où $d\sigma^2$ est de la forme (3), mais n'est plus euclidien (à 3 variables).

On sait (de par la corrélation entre le principe d'Hamilton et le principe de la moindre action) que les trajectoires de P sont alors données par les géodésiques du ds_1^2 à trois variables $ds_1^2 = (U + h)\,d\sigma^2$, h constante arbitraire, et t par $dt = \frac{U\,d\sigma}{\sqrt{U+h}}$. Les trajectoires de la lumière s'obtiennent en faisant $h = 0$, d'où alors $dt = \sqrt{U}\,d\sigma$.

Painlevé generalizes his geometric Newtonian form to general relativity

This generalization is correct in the case of null geodesics (light). This induces that the space-time defined by general relativity and the generalized Painlevé geometric Newtonian form have the same conformal structure which, as we know, governs causality.

But, it fails to describe the timelike geodesics identical to those of general relativity, so does not describe the same spacetime.

The Newtonian and relativistic theories are different, epistemologically based on different assumptions which, except in some boundary weak field conditions, give different results.

This exhibits the trouble they had with theconcepts of space and time, such as defined by the general relativity.

The case of the single body with spatial spherical symmetry exhibits pseudo-Newtonian characters

The relativistic gravitation takes into account the self-interaction of a gravitational mass of a body with itself (its active mass with its passive mass), in contrast to Newtonian theory.

In the case of a single mass, given the spherical symmetry, this phenomenology can be obscured because the active mass in general relativity is supposed to be equal to the Newtonian gravitationnal mass at infinity.

In this solution, for radial free fall, without initial velocity at infinity, general relativity and Newtonian mechanics are equivalent.

Let's recall that mass is divided into three categories: inertial mass and the gravitational active and passive masses. The inertial mass and passive gravitational mass are equal according to the equivalence principle. The gravitational active mass contributes (with all the others) to the gravitational field and all the passive masses, including its own, couple with this global gravitationnal field.

Outstanding communication to the Academy of Sciences: Most of these masterful contributions will be forgotten!

From 1921 until 1924, a debate, polite but harsh, is raging at the Academy of Sciences between supporters and opponents of relativity, with on a side Le Roux, the most active defender of classical mechanics and on the other Brillouin, leader of defenders of Einstein's theory.

Alongside this fierce debate, let us notice 3 innovative contributions.

Sauger (1922) establishes the Schwarzschild solution directly from special relativity.

Cartan (1922) discovers the null principal directions of such a space-time, foreshadowing the classification of Petrov (1954)-Pirani (1956).

Chazy (1922) derives, the Schwarzschild solution with cosmological constant. That will be rediscovered independantly by Lemaitre (1932).

Conceptual difficulties of general relativity

The integrated concept of space-time is in stark contrast to what we have in mind where space and time perceived as independent entities.

The concept of intrinsic curvature of four-dimensional spacetime and even of three-dimensional space is difficult to represent.

The absence of an absolute background space, does not facilitate the understanding of orientation and space motion.

The hyperbolic spacetime is conceptually difficult to represent. Most 2D graphs we use for illustration are geometrically false.

The physical character of a geometric representation of the gravitation, hence relying on properties of space-time, is not well accepted by all.

Scientists thought wrongly that the symmetry of the Einstein's equation, would imply the “reversibility” invoked by Painlevé!

What about the difficulty to think “covariant” ?

To face this conceptual difficulties, one has tried to use methods separating space and time in the analysis (ADM, pseudo-tensor, ..).

The ADM approach was motivated by the emergence around 1960, of numerical methods for solving equations such as Einstein equations. It is, independently, also a method paving the way for a possible quantification of a theory of gravitation.

Meanwhile, some effective covariant approaches have also been developped:

Classification of Petrov-Pirani, prefigured by E. Cartan.

Congruences can reconcile approaches deemed "incompatible", because they rely on classes of four-dimensionnal geometric objects. Therefore, as the physical world is four-dimensionnal, they can be tested.

The difficult emergence of new ideas

The example of Painlevé shows how a great solution, can result from a misinterpretation.

This highlights the phenomenon of the emergence of theories of rupture, which can not be directly derived from the existing ones. It should not be surprising that the authors can be scientists "out of their field of excellence."

The history of relativity shows that the "discoverers" have not always been aware of what they had found and, in case, they did not often realize their importance and rarely have measured their scope.

From what seems to be a detail (generalization of Minkowski's geodesic), Einstein built, on this *phenomenological argument*, a monument. This shows the power of some heuristic arguments!

The controversy about Painlevé assertions on the ds^2

We showed how Painlevé's assertions on ds^2 were misunderstood. One may wonder how such misunderstanding was possible as it was so simple to explain.

The context of "nervousness" of the proponents of the new theory still highly contested and plagued by internal problems (problem of "infinite potential on the horizon") was not favorable to a peaceful debate.

This led to a disaster for the scientific community.

What lessons can be learned to improve the sustainability of such work?

Conclusion: Painlevé an exemplary case showing the difficult emergence of new paradigms

It is surprising that it is Painlevé, who is a scientist educated in the classical Newtonian theory, who opens up an innovative debate on foundations and epistemological implications of the general relativity.

One said that Painlevé was a mediocre relativist. Whether we refer to his understanding of the theory, clearly, this is true. But, notwithstanding with his poor skill in this theory, he set up an innovative form of the metric who has baffled even the most brilliant minds, including Einstein, and whose merits are, at last, recognized today.

One may consider this as a happy coincidence, but one may also consider that his poor understanding of the general relativity prevented him to stick at already approved concepts and allowed his mind, free of these constraints, to be open at new ideas.