E = Mc^2: The Sun energy

"The influence of the Sun on the Earth, from which all movement and all life, all order and all ornament of nature derive, is such that the more we consider it, the more we find it wonderful. Hence for the philosopher the commitment to use all the resources of his mind, in order to rise himself to a theory worth of such subject. " *Johannes Kepler (1571-1630).*

$E = Mc^2$

A demo is provided in this presentation

Jacques Fric - Société Astronomique de Montgeron –updated 14/03/2022

$E = Mc^2$

In this well-known formula, valid locally, one easily understands that the matter's energy E is proportional to its mass, but one may be surprised by the factor including the celerity of light.

Per the high value of the celerity of light, this shows that this energy of matter, which is a potential energy, is huge, something that nobody expected before the twentieth century.

This equation, implied by the special relativity, will be laboriously established by Einstein in this framework His first controversial demonstration in 1905, suffers from only being valid in the case of a velocity v such that $v/c \ll 1$.

$E = Mc^2$

Shortly after the publication of his first article in 1905 on special relativity that few people had understood, in the same year, Einstein published a second article where he proposed a demonstration of this equation disputed by some. See references for these articles. His demonstration, see references [3] [4], is reproduced in appendix A-1

He will give several others. We will present a demonstration of Yves (1952), see reference [7], cited by [1] and [8], which is not by Einstein, but which is general and simpler.

$E = Mc^2$

This demonstration will show that the presence of the term in c^2 (speed of light squared) is indeed a consequence of the theory of special relativity, in particular of its fundamental principle which is that all the laws of physics must be the same in all inertial frames, also called Galilean frames, in homage to Galileo.

But, for that, we must start by considering the quantities which appear in the equation because, if we have an intuitive idea of what is mass and energy, when we look more closely, we note that it is not as simple as it sounds.

WHAT IS MASS?

This concept which seems familiar to us is not as simple as it seems.

Indeed, there are three types of mass

Gravitational mass

The gravitational mass is divided into 2 categories:

1 - Passive gravitational mass

Commonly called weighty mass, a term that we will adopt, we will consider it briefly, although it does not intervene in the development of the equation $E = Mc^2$, because it played a role in our appreciation of the concept of mass.

2 - The active gravitational mass

The active gravitational mass generates the gravitational field. It is at this active mass of the Earth, for example, that the weighty mass of an object couples for generating its weight.

Passive gravitational mass

This property is exhibited in an obvious way by the "weight" of material objects: On Earth, to lift an object lying on the ground, it is necessary to make an effort proportional to the quantity, depending on its volume, of matter (of the same kind).

But this weight, for a given volume of matter, depends on the matter. Wood, iron, lead, etc. obviously have different weights for the same volume.

This mass is therefore a property of matter since it depends on it.

Inertial mass

In a more subtle way, another character corresponding to a different phenomenology exists: It is the "resistance" of bodies to the change of their relative velocity to a given reference.

Thus on Earth, on a smooth horizontal surface, to set in motion a massive sphere for example, it is necessary to apply an effort and this, during a time depending on the desired change of velocity.

If the sphere, of the same material, is twice as large, it will be necessary to apply a force twice greater, to get the same result. Again the effort is proportional to the amount of matter of a given nature.

Inertial mass

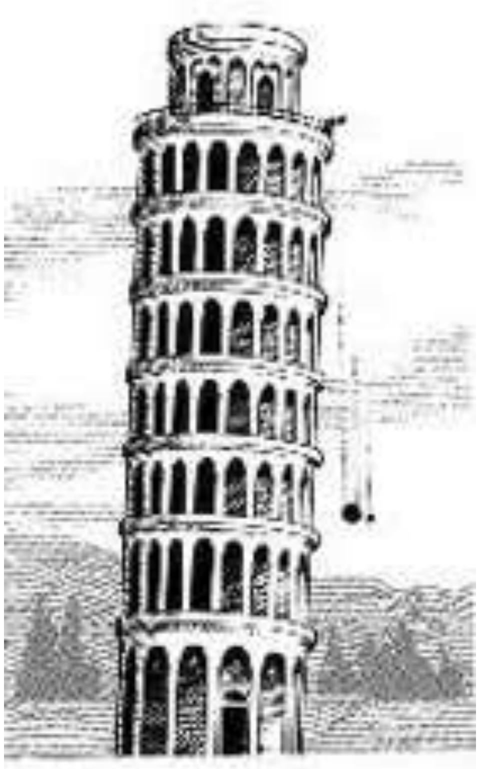
This property of mass that resists to change is called inertia. Therefore this defines an "inertial mass". In the equation $E = Mc^2$, M is the inertial mass.

The body set in motion keeps its velocity when the applied force ceases (neglecting friction).

Such frame, with relative constant velocity to a reference inertial frame, is called inertial. The concept of inertia is a fundamental concept in classical mechanics and in relativity.

An essential property of an inertial frame is that it is a frame where no constraint is applied on the extended bodies which are lying in it (free fall).

Equivalence principle



The concept of weighty mass and the concept of inertial mass which seem different are closely linked by what is called the principle of equivalence whose experimental observation is attributed to Galileo.

From the top of the tower of Pisa, two bodies of different masses, released at the same time, touch the ground at the same time.

The gravitational field of the Earth would confer an acceleration to the more massive body greater than that of the less massive body, but this is exactly compensated by their inertial mass, whose resistance to change (acceleration) of the more massive body is stronger than the other.

Equivalence principle

The fact that the movement, in free fall, does not depend on the mass of the object shows that the weighty mass and the inertial mass are equal.
This is the Galileo equivalence principle (for gravitation), which Einstein extended to general relativity.

A material object simultaneously has the 3 mass attributes: It has an inertial mass, a passive mass and an active mass.

As the strength of the active mass is very small, huge masses are necessary for being not neglected. We will not consider it here.

Let’s add that in classical mechanics the passive mass of a body does not couple with its own active mass unlike in general relativity.

Mach‘s principle

The principle of equivalence will give rise to a "gravitational" interpretation of the inertial mass by Mach: The inertial mass of a body results from the interaction of said mass with all the masses of the universe.

This is corroborated experimentally by the phenomenology of inertial equipment, like the pendulum, the gyroscope, which do not seem (or very little) to depend on the close masses but indeed of the whole universe.

Thus a pendulum on Earth launched in one direction will keep it, notwithstanding the rotation of the Earth and the influence of the Sun and the galaxy.

Kinematics definition of mass

The Mach's principle would imply that in an « empty » universe the bodies would have no inertial mass. This bothered Einstein which was led to discard this principle from his analysis.

In Newtonian mechanics, the inertial mass *m* is defined by the equation $f = m.\gamma$, where *f* is the force and γ the resulting acceleration of the body of masse *m*.

One may also define the inertial mass *m* by the preservation of momentum, in elastic collisions ($\boldsymbol{m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2}$) where $\boldsymbol{m_1}$ is the mass of body 1 of velocity $\boldsymbol{v_1}$ before collision and $\boldsymbol{v'_1}$ after and $\boldsymbol{m_2}$ the mass of body 2 of velocity $\boldsymbol{v_2}$ before collision and $\boldsymbol{v'_2}$ after. This relation shows that the ratio of change in velocity ($\boldsymbol{v_1 - v'_1}$) and ($\boldsymbol{v_2 - v'_2}$) is inversely proportional to the masses $\boldsymbol{m_1}$, $\boldsymbol{m_2}$, this allowing, from a known mass, to define all the other masses.

What is energy?

Modern theories show that this concept which looks familiar is in fact the physical representation of time

Energy

Energy is a vital element of which we have a fairly external idea, through the consumption that we make of it which results from a transfer from a source with potential energy, to a destination, from the power station to the vacuum cleaner, for example through the electrical network and household outlet.

There is a general distinction between potential energy and kinetic energy.

In relativity, energy is the time component of the 4-momentum vector. It is the physical quantity associated with time.

In quantum mechanics, the energy E, in the Schrödinger equation is associated with the operator $i\hbar\partial t$ acting on the wave function ψ. Energy is the measure of its variation over time (see A-3).

Energy

This shows that the equations, of the two fundamental theories of modern science, relativity and quantum mechanics associate energy with time.

The meaning of this is that without energy transfer, physical time does not flow.

This energy transfer is involved, among other things, in our metabolism, the flow of time that we perceive is only possible because energy in our body is consumed.

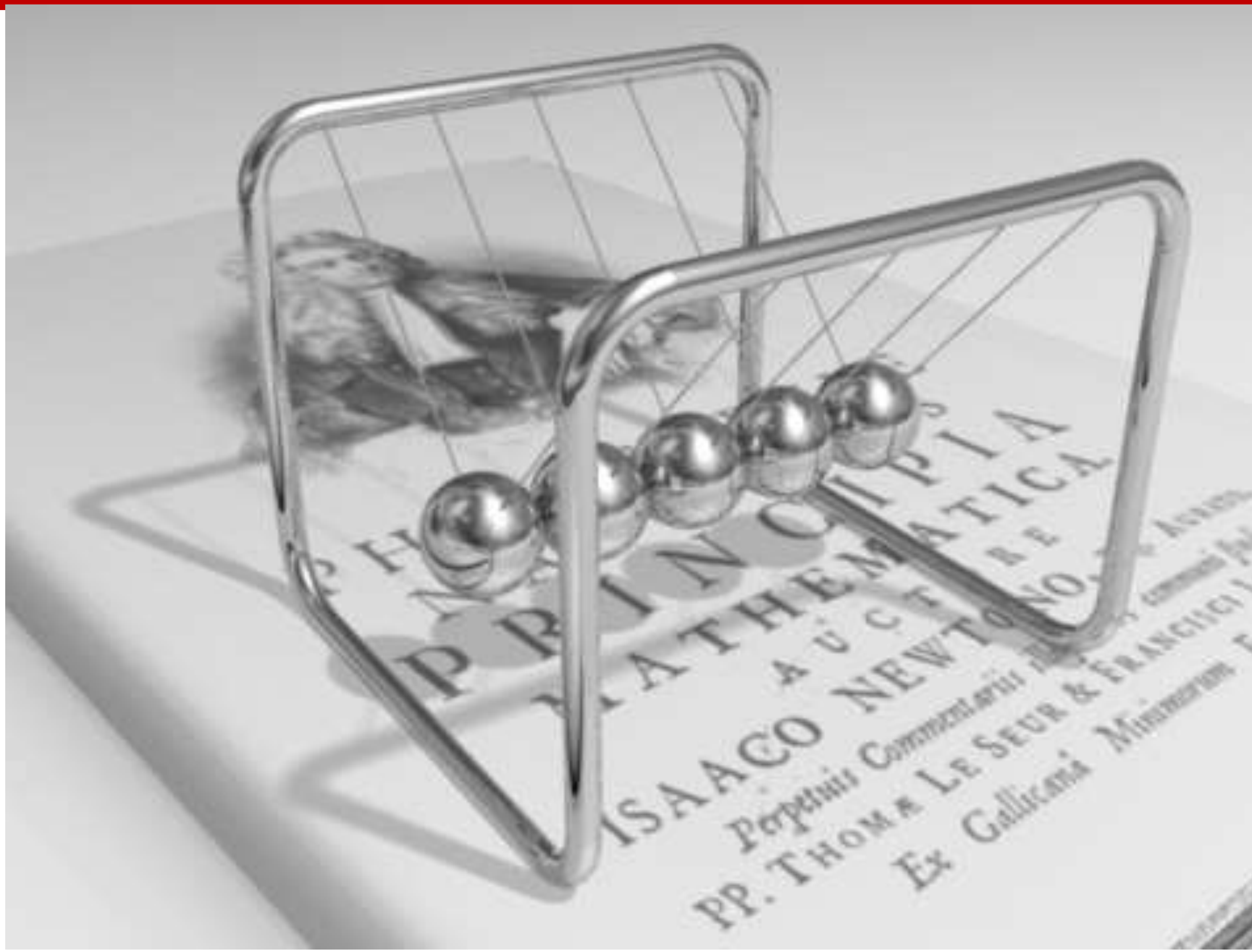
The relativistic momentum

In classical physics the momentum noted P is a spatial vector equal to the product of the mass m of a body by its velocity vector v, with respect to the observer who measures v: $P = mv$.
The conservation laws of this quantity are linked to the invariance of physics with respect to the choice of a position, according to the theorem of E. Noether (which also applies in relativity):

An isolated system has a constant momentum. If internal forces split it into parts, the sum of the parts' momentum equals its initial momentum.

In relativity, space-time theory, we cannot keep the Newtonian form, we must use instead: $P = \gamma mv$, formula that we will use, where γ, Lorentz factor, introduces the effect of time. See appendix A-4: Proof of this formula and an example. .

Momentum



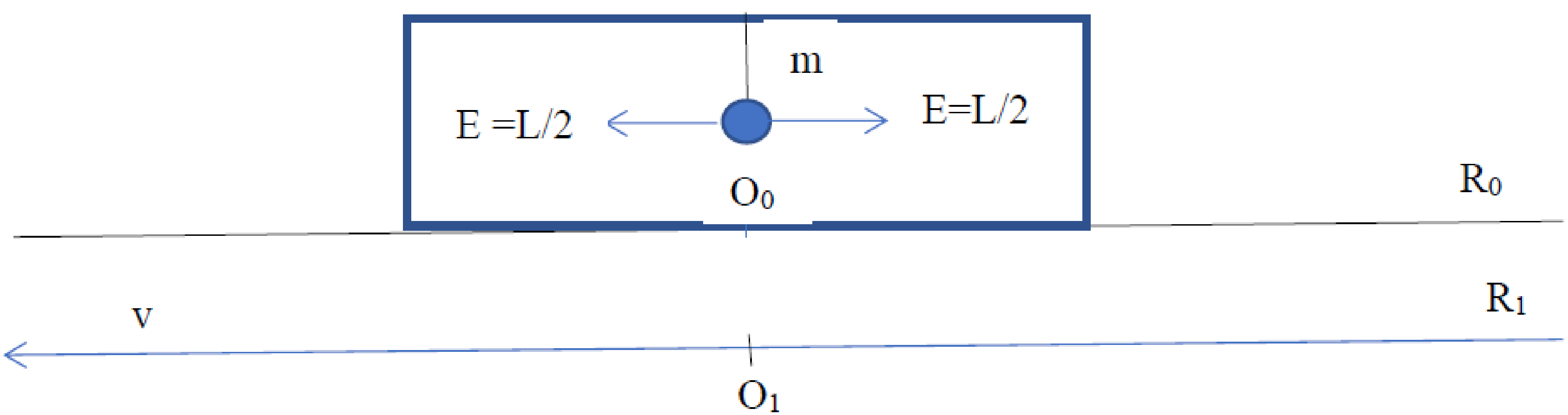
Newton's pendulum illustrates the conservation of momentum

$E = Mc^2$: A demonstration

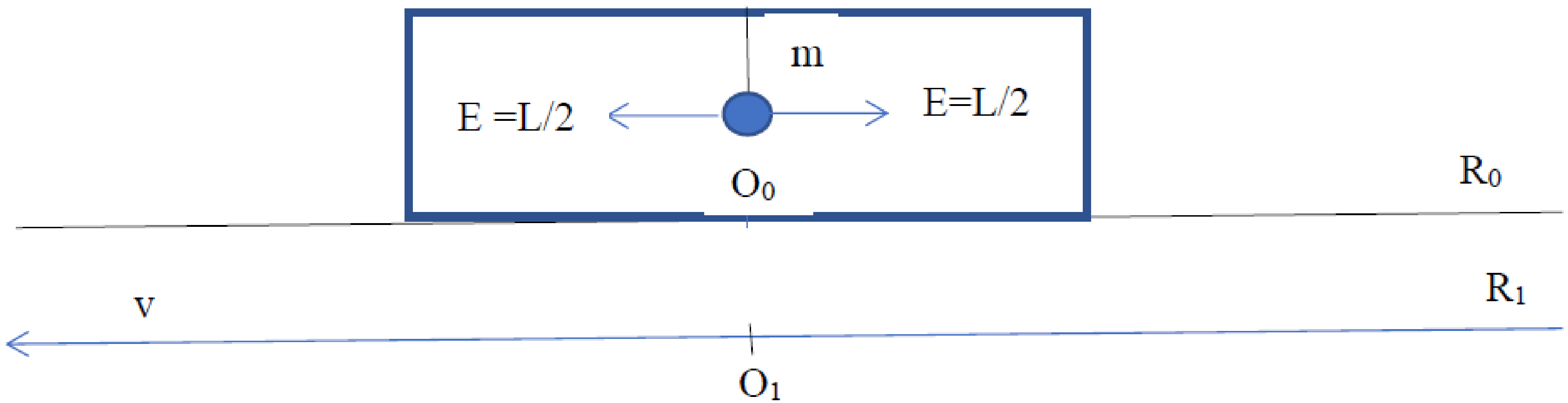
In the context of special relativity, let us describe a demonstration, given by Yves in 1954, based on the same thought experiment, that Einstein used in his article of 1905.

A body of mass *m* is hanged into a box by a non-conductive wire. Suddenly it emits two light pulses (photons) of energy $E = L/2$ in two opposite directions. First, let us consider the point of view of an observer O_0 located in the reference frame R_0, where the box is located.

Let us recall that in relativity the mass *m* is an invariant 4-scalar.



$E = Mc^2$: A demonstration



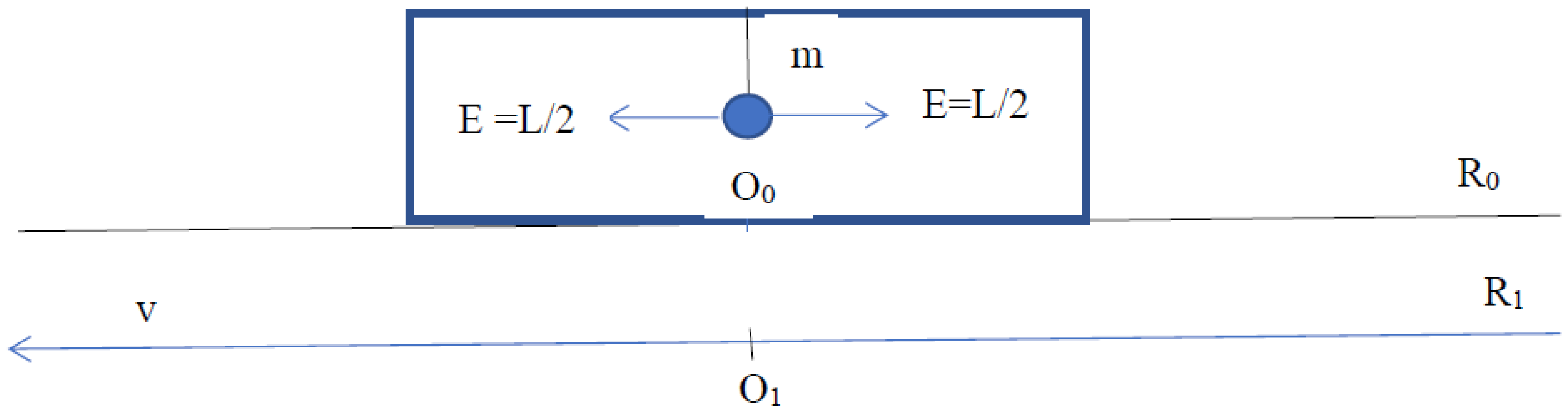
The momentum of the photon emitted to the right is $p = L / 2c$, that of the photon emitted to the left is $-L / 2c$. The momentum P of a photon of energy $E = L$ is $P = L/c$ This was demonstrated by Maxwell (and later Poincaré).

The conservation of the total momentum P implies that the body remains at rest in the box, therefore:

In R_0, before the emission: $\boldsymbol{P} = \gamma m v = 0$ because $\boldsymbol{v} = 0$

Where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the well-known Lorentz factor.

$E = Mc^2$: A demonstration



In $\boldsymbol{R_0}$, after the emission , we get :

$$\boldsymbol{P} = m'\boldsymbol{v} + \boldsymbol{L/2c} - \boldsymbol{L/2c} = 0$$

Because $\boldsymbol{v} = 0$.

Where the mass of the body, after the emission is denoted m'.

The idea, for linking mass to energy, will be to evaluate the loss of mass $m'-m = \Delta m$ of the mass m, when it loses an energy L.

$E = Mc^2$: A demonstration

The laws of physics must have the same form in all inertial frames! This does not mean that everyone will get the same results, this is not the case, but that these results will obey the same laws: Here the law is the conservation of total momentum.

In a 2nd step, let's take the point of view of an observer O_1 in an other Galilean frame R_1 moving at constant celerity v, to the left, relative to the box.

We will use an equation, given by Einstein in 1905 which describes the relativistic transformation of the energy E of light rays, which is written:

$$E' = E\left(\frac{1 - \cos(\theta)\frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}\right)$$

$E = Mc^2$: A demonstration

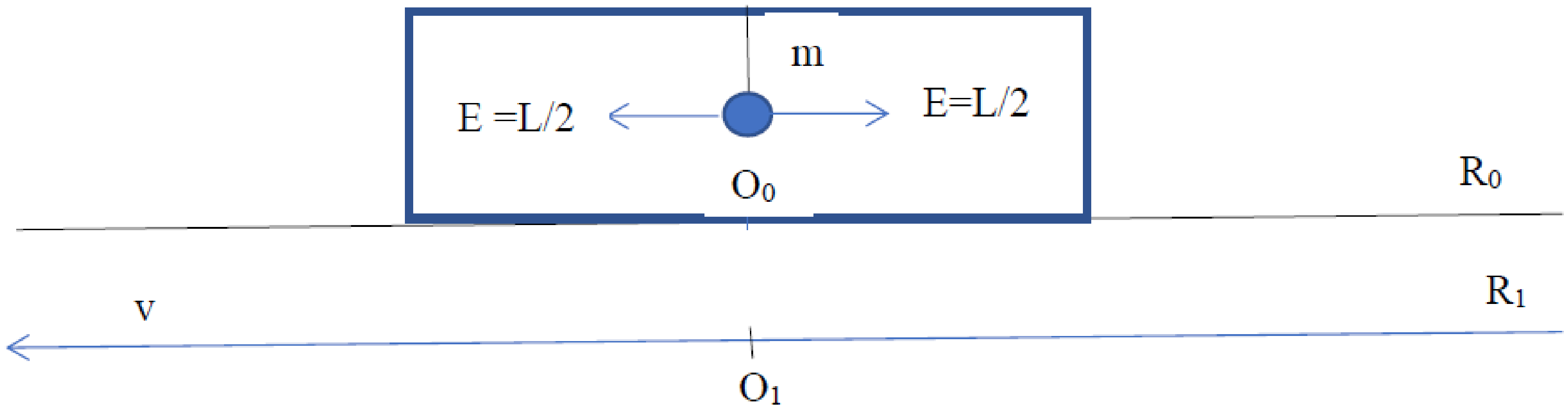
Where E is the energy of the light ray in the inertial frame of reference R_0 where it is emitted and E' is the energy measured in the frame of reference R_1 moving with a constant velocity v relative to R_0 and making an angle of θ with R_0. This is simply deduced from the relativistic Doppler effect, the energy of a photon being proportional to its frequency.

In the Galilean frame $\boldsymbol{R_1}$, the total momentum conservation of the system can be written:

$$\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m'v}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{L}{2c}\frac{1-\frac{v}{c}\cos\pi}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{-L}{2c}\frac{1-\frac{v}{c}}{1-\frac{v^2}{c^2}}$$

Momentum of the body before emission	Momentum of the body after emission	Momentum of the right emitted photon	Momentum of the left emitted photon

E = Mc²: A demonstration



In the above figure, $\theta = \pi$, for the photon emitted to the right (direction of the photon opposite to $\boldsymbol{v}$) and $\theta = 0$ for the photon emitted to the left (same direction than $\boldsymbol{v}$).

$E = Mc^2$: A demonstration

$$\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m'v}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{L}{2c}\frac{1-\frac{v}{c}\cos\pi}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{-L}{2c}\frac{1-\frac{v}{c}}{1-\frac{v^2}{c^2}}$$

As the symmetrical emission of photons leaves the box at rest in R_0, its relative velocity *v* in R_1 does not vary and as ***cos(π) = -1,*** we get:

$$\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}(m-m') = \frac{L}{2c\sqrt{1-\frac{v^2}{c^2}}}\left(\left(1-\frac{v}{c}(-1)\right)+\left(-\left(1-\frac{v}{c}\right)\right)\right)\rightarrow$$

$$\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}(m-m') = \frac{L}{2c\sqrt{1-\frac{v^2}{c^2}}}\left(2\frac{v}{c}\right)\rightarrow \frac{v}{\sqrt{1-\frac{v^2}{c^2}}}(m-m') = \frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\frac{L}{c^2}$$

$E = Mc^2$: A demonstration

Symplifying last equation yields:

$$(m-m') = \Delta m = L/c^2$$

As the total energy ***E*** emitted by the 2 photons is **$L/2 \times 2 = L$**, the missing energy corresponding to the loss of the mass Δm is :

$$E = \Delta m \, c^2$$

We note that it is because the phenomenon must satisfy the rules of special relativity (the laws of physics are the same in all inertial frames) that the term c^2, appears in the formula.

Of course this result of special relativity is also applicable, locally, in general relativity.

$E = Mc^2$: A demonstration

This demonstration has the advantage of relying on a thought experiment. There are some of this type, but this one is the simplest and above all the most explicit.

Today we do not demonstrate this formula in this way. The general form of which is:

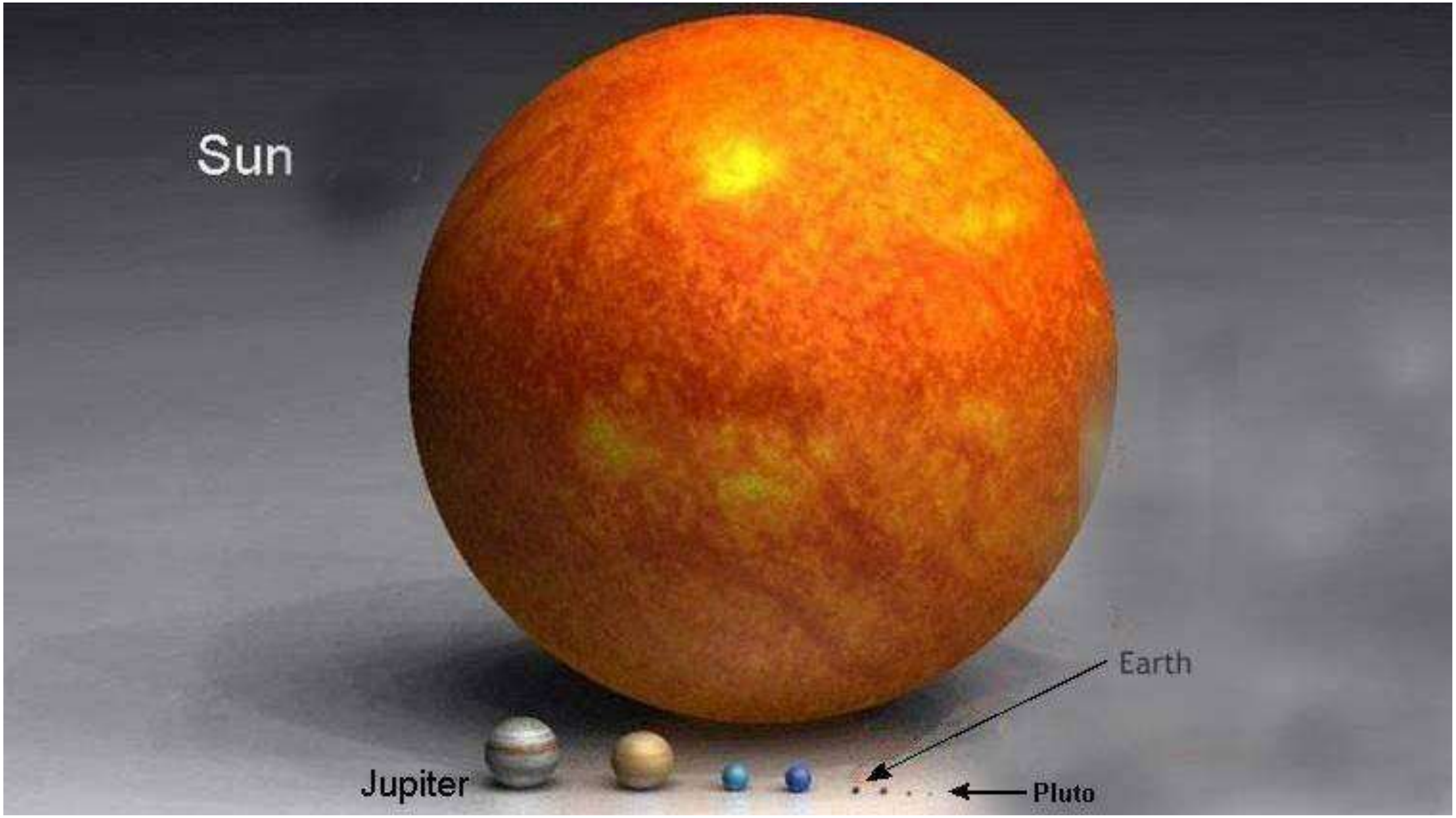
$$E^2 = m^2c^4 + p^2c^2$$

Which reduces to $\boldsymbol{E = mc^2}$ when $p = 0$ which occurs when $v = 0$, per the definition of p.

.

A modern demonstration is given in annex A-2

The Sun is a star as these that we can see in the night, but far more close than them.



It is a huge ball of gaz (mainly hydrogene) whose hart is in nuclear fusion. Its size is gigantic, its radius is 100 time larger than that of Earth.

The mystery of the energy of the Sun

What is the power of the Sun ?
How long will it shine before it runs out of fuel?
What fuel does it use?
How long has it been shining?

At the beginning of the 19th century, coal was naturally considered, but that would only provide such energy for a few thousand years.

The first to seriously consider these questions was the great German physicist Hermann von Helmholtz who emphasized, in 1854, that the Sun's gravity, alone, could provide such energy during about 20 millions of years.

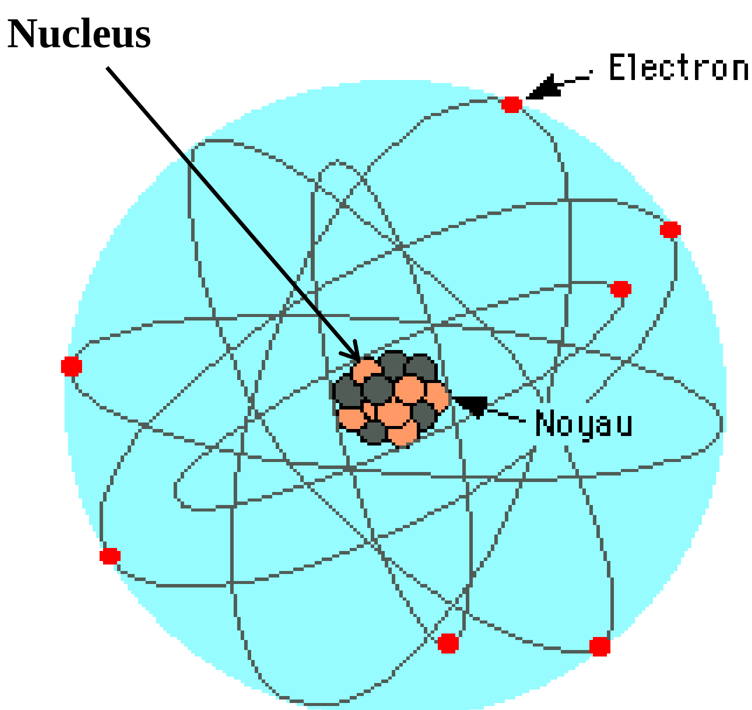
The mystery of the energy of the Sun

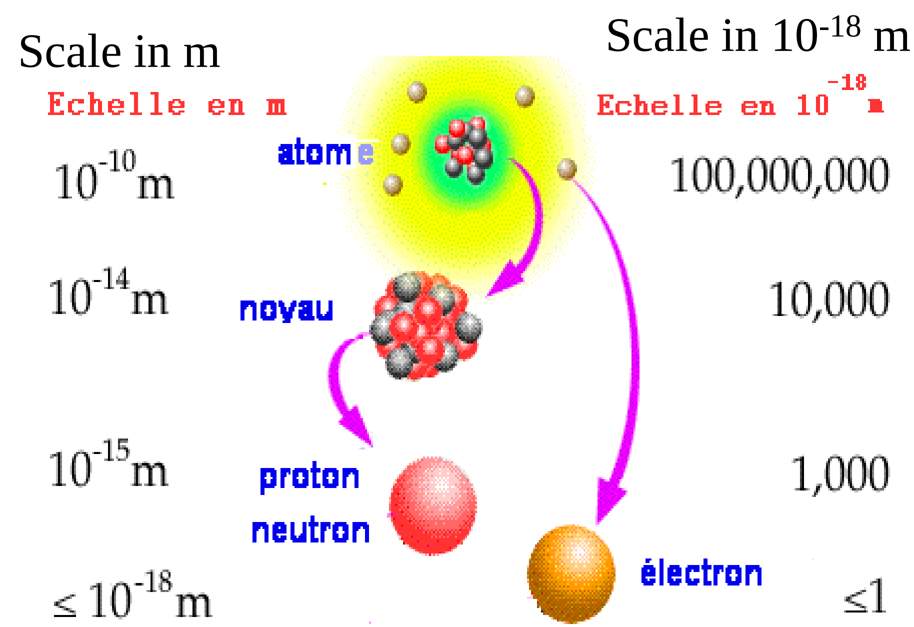
Radioactivity was then discovered:

Radioactivity made it possible to estimate, that the Earth is several billion years old, a much greater age than that given by Helmholtz, and that this source of internal energy in the nucleus of the atom, newly discovered, could supply the energy of the Sun.

Fission was considered first, but later, the chemical composition of the Sun led to consider the fusion.

Atoms





Schematic representation of an atom with its nucleus and its electrons, the scale is not respected, the size of the atom being 100,000 times larger than that of nucleons.

Atom's nucleus

Protons and neutrons, generically called nucleons, form an atomic nucleus of femtometric dimension, 10^{-15}m

As the protons are positively charged, they encounter a repulsion within the nucleus, but the intensity of this electrostatic repulsion is less than that of the attraction between nucleons induced by the "nuclear" force at distances less than 2.5 fm.

As neutrons are not charged, they do not encounter a repulsion within the nucleus, but just the attraction induced by the "nuclear" force. Therefore they will improve the cohesion of the nucleus for all atoms, other than hydrogen.

Note that it is not directly the strong interaction that is at work because it is exerted between the quarks, but likely a residual effect called the "nuclear" force which is not a fundamental interaction.

Cohesion of atom's nucleus

Before discovering quarks, this interaction that links nucleons was called nuclear force. It is not a fundamental interaction. It can be interpreted in terms of exchanges of light mesons, like pions, unlike the strong interaction between quarks which is interpreted by exchanges of gluons.

The cohesion of the nucleus therefore results from a binding energy between the nucleons. Energy must be supplied to separate the components of the nucleus. Conversely, when we are going to merge nucleons to form a heavier nucleus, this binding energy will be released. To be stable, the nucleus must have an energy lower than that of the sum of these unbound (free) constituents.

The nuclear fusion energy is not the energy of the nucleon matter ($E = Mc^2$), but it is the binding energy between nucleons.

(Thermo)Nuclear fusion

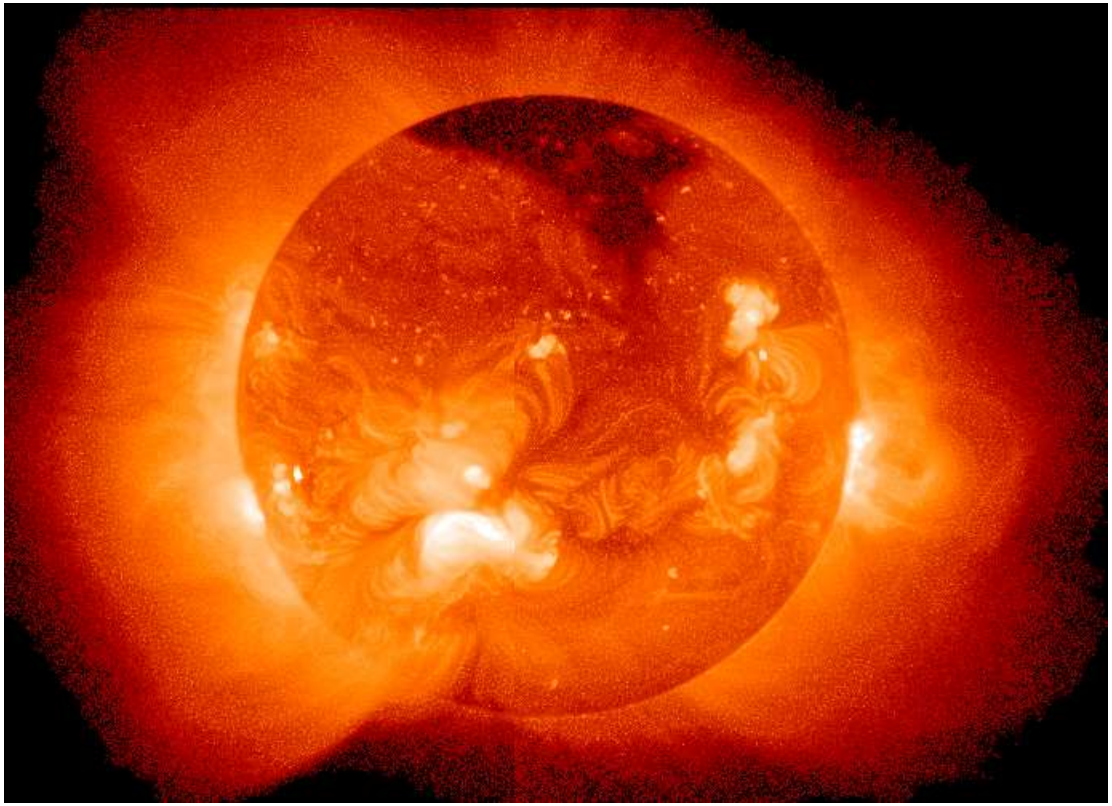
Nuclear fusion, sometimes called thermonuclear fusion, is a process in which two atomic nuclei come together to form a heavier nucleus.

This reaction is at work in a natural and long stable way, in the Sun and most of the stars of the universe.

The fusion of light nuclei gives off enormous amounts of energy from the binding of nucleons due to what has been called "nuclear force".

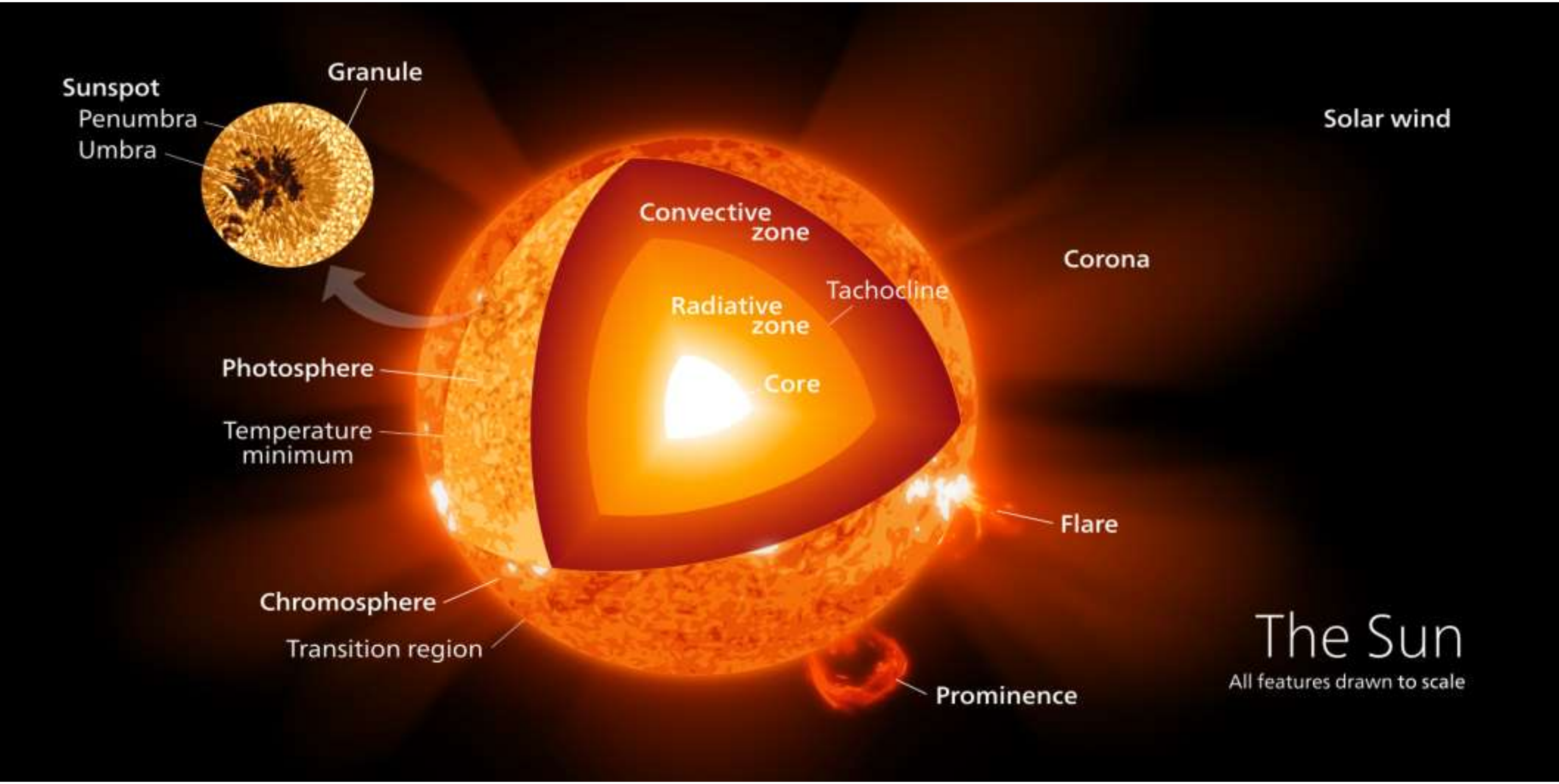
It is, with fission, one of the two main types of nuclear reactions applied. One of its advantages is to be able to theoretically produce much more energy (from 3 to 4 times more), with an equal mass of "fuel", than fission.

The nuclear fusion source of energy of the Sun

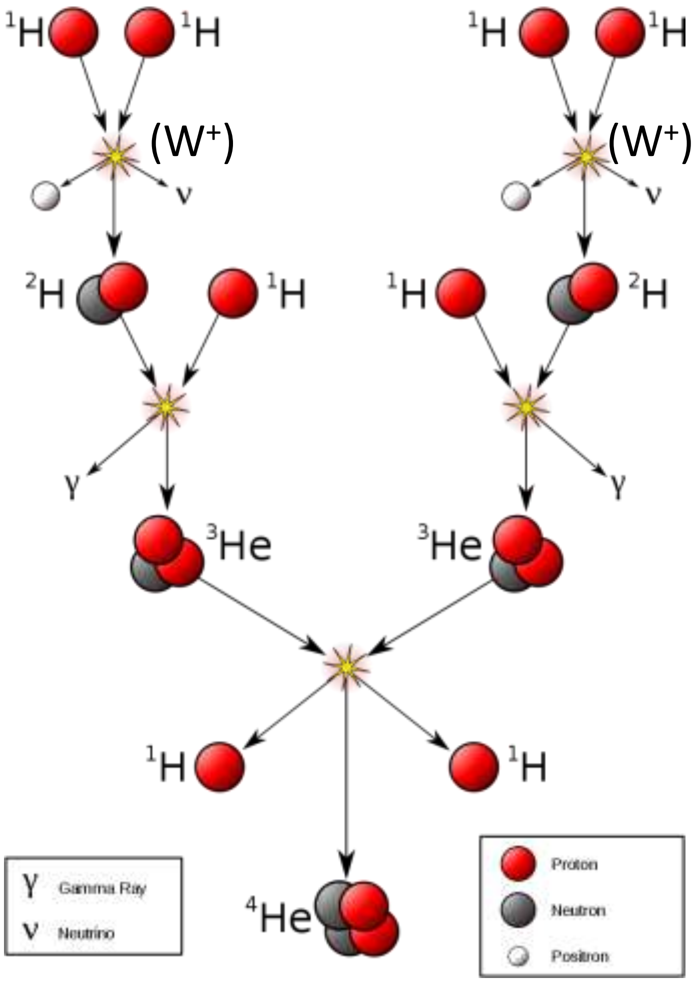


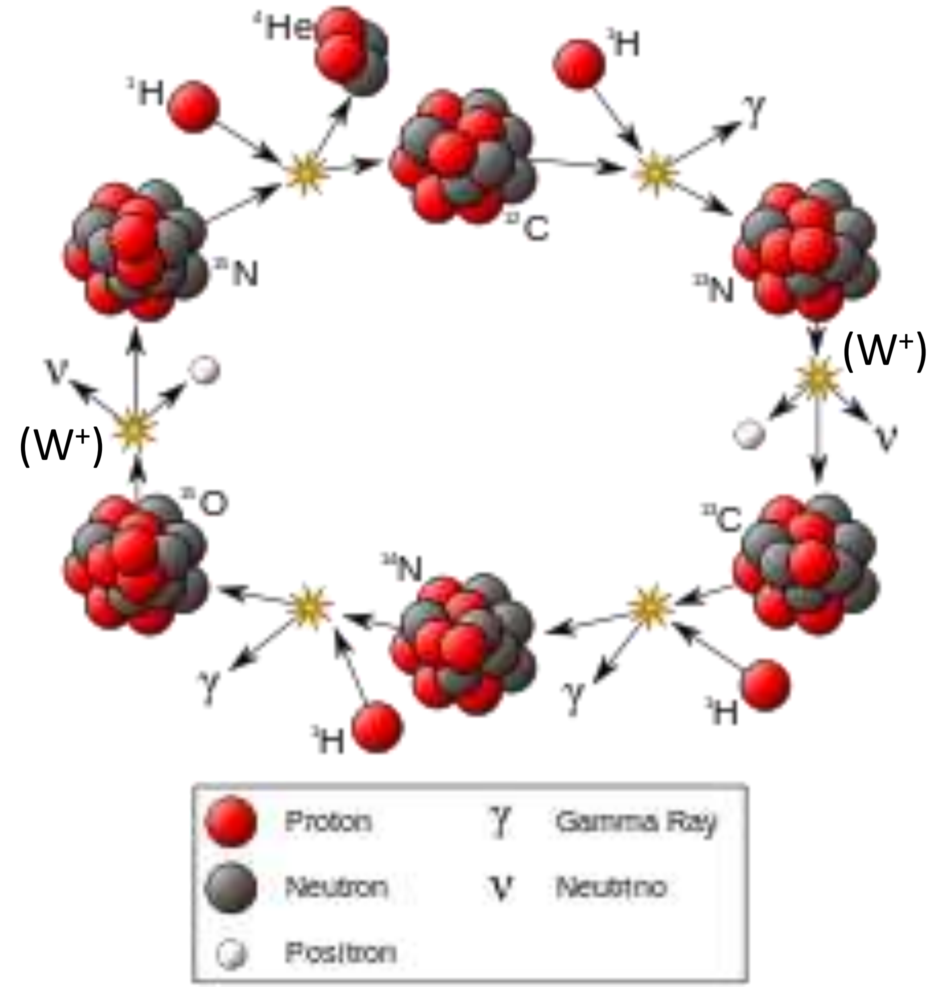
The Sun generates many nuclear reactions of fusion.

Internal structure of the Sun



Nuclear reactions in the Sun





The **weak interaction** allows the proton-proton fusion which predominates in stars of the same size or smaller than that of the Sun.

The carbon-nitrogen-oxygen cycle predominates in stars with mass greater than that of the Sun.

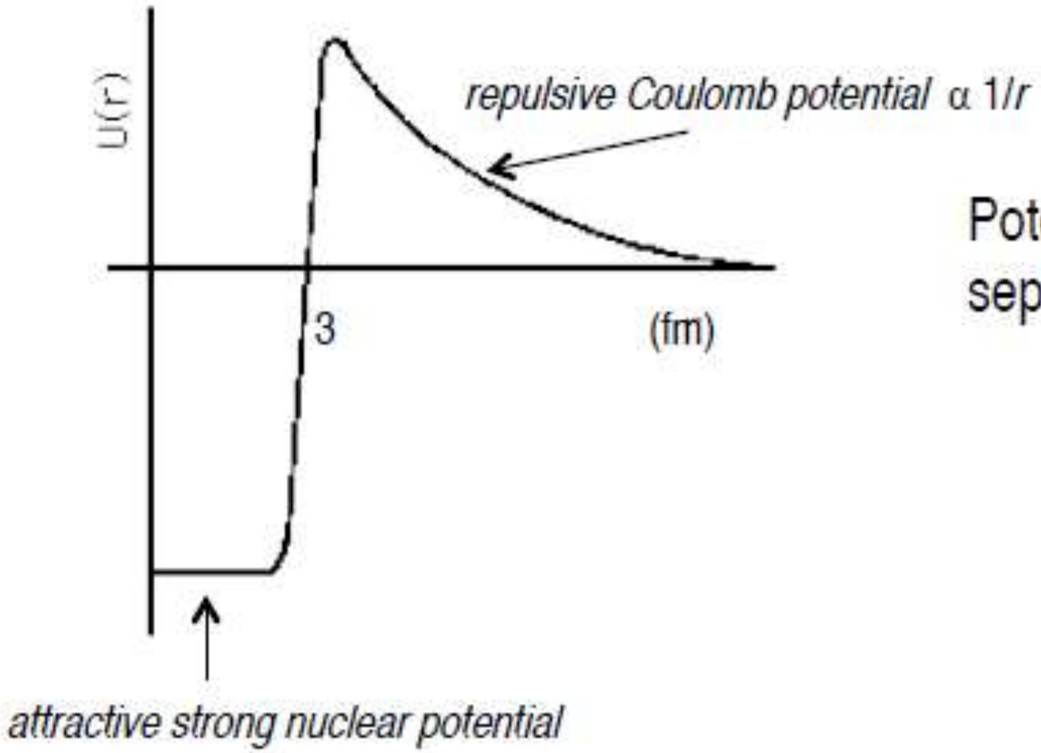
Nuclear reactions in the Sun

Interaction between protons

A very high temperature, i.e. a very high kinetic energy, is needed, because these reactions only can occur, in the reaction above, when 2 protons are close enough to merge. For doing that they must be able to cross a very high potential barrier, because they repel each other, being both positively charged.

Indeed if the "nuclear force" is at short distance greater than electromagnetic repulsion this difference is not very large, as evidenced by the fact that Helium 2 (2 protons only) is not stable.

Nuclear reactions in the Sun



Potential energy versus nuclei separation.

To merge, two positive charges must cross the potential barrier of the electric field of infinite range, whose repulsive force increases in $1/r^2$, to reach the border of the potential well, where the nuclear force dominates (typically 3.10^{-15} m, which is 3 times the size of the nucleus).

Nuclear force

The nuclear force, which is exerted between nucleons, is responsible for the binding of protons and neutrons in atomic nuclei. It can be interpreted in terms of exchanges of light mesons, like pions.

Even if its existence has been demonstrated since the 1930s, scientists have not succeeded in establishing a law allowing its value to be calculated from known parameters.

It is sometimes called residual strong force, to distinguish it from strong interaction.
This formulation was introduced in the 1970s due to a change of paradigm.

Previously, the strong nuclear force designated the force between nucleons.

Nuclear force

After the introduction of the quark model, the strong interaction designated the forces defined by quantum chromodynamics, which interact with quarks, due to their color charge.
Since nucleons have no color charge, nuclear force therefore does not directly involve gluons, the mediators of the strong interaction, but rather other processes.

Basic properties

Nuclear force only involve hadrons.
At typical separation distances of nucleons (1.3 fm), it is a very intense attractive force.
At these distances, the nuclear force is more intense than the Coulomb force. It can therefore overcome the repulsion between protons produced by the Coulomb force inside atomic nuclei.

Nuclear force

However, the Coulomb force between protons has a greater range and becomes the only significant force between protons when they are separated by more than 3 fm. On the other hand, at very short distances, the nuclear force becomes highly repulsive, which maintains a real separation between nucleons.

The nuclear force (NN force) is practically independent of the type of nucleons (neutrons or protons). This property is called charge independence.

The NN force depends on the relative orientation of the spins of the nucleons which can be parallel or antiparallel.

The force NN has a non-central or tensorial component. This part of the force does not retain the orbital angular momentum, which is a constant of the movement produced by a central force.

Nuclear reactions in the Sun

The essential role of neutrons

On the other hand, the capture of a neutron does not undergo this repulsion and on the contrary it is favored for low energy neutrons.

Neutrons play the role of glue between the nucleons. Without them there would be no nucleus other than hydrogen and no chemistry, especially organic, so we would not be here.

It was the rescue, in extremis, of 1 neutron out of 7, after 200s of carnage, during the primordial nucleosynthesis, which allowed the adventure to continue up to human being .

Cosmological recall

This thermonuclear fusion in the stars is possible because the primordial nucleosynthesis, where it was the whole universe which was in fusion, left enough hydrogen so that the stars can, much later, locally and more modestly, finish the job up to human beings.

If the nucleosynthesis had gone well, all the hydrogen would have been burnt up to iron : End of story!

Nucleosynthesis could have started around $t \approx 1s$, (all times are given in cosmological time), where the energy of the universe was 1 Mev (10^{10} ° K), because the binding energy of the nucleons is of this magnitude. This was not the case for several reasons.

An upset nucleosynthesis

1-The universe was expanding very quickly: Not easy to meet in an environment that is fleeing apart in all directions..

2-Deuterium, which is an essential link in the chain, has a fragile core. It can be easily destroyed by photons: This is called the deuterium chicane.

3- Precisely photons, there are one billion per nucleon, following the annihilation of matter - antimatter, which occurred at a temperature T = 1 GeV (at t = 10^{-6} s) where only 1 billionth of matter in excess, in violation of the laws of physics, has survived.

At t = 1s, the thermal equilibrium at 1 MeV follows a statistic where there are more photons with energy over 1 MeV, than nucleons. This allows them to massively destroy the deuterium nuclei, whose binding energy is only 1 MeV.

t = 100s, the deuteriun is stable

It will be necessary to wait until 100 seconds before nucleosynthesis can be effective. The temperature is then 100 KeV, the universe expands more slowly, and the photons are less energetic. Deuterium is no longer massively destroyed: The fusion of hydrogen to helium becomes effective.

But it is very late, and, at this temperature, the fusion of hydrogen becomes sluggish and finally, at t = 200s, this fusion will stop. The result is roughly 10% of helium synthesized. But this helium will incorporate and save all the neutrons that survived (1 out of 7) from previous episodes.

After primordial nucleosynthesis, the universe is made up of 90% hydrogen nuclei and 10% helium nuclei and traces ($<10^{-7}$) of other light elements.

The cosmos did the job

By leaving 90% hydrogen, the universe preserves the future: Large structures generating stars with their processions of planets will exist.

By fusing the neutrons in the helium nuclei which stabilizes them, it saved the surviving neutrons which allows complex and varied chemistry (All nuclei except hydrogen require neutrons).

Note that it is the asymmetry of probability of the reactions ($A = p \rightarrow n + e^{+} + \nu$) and ($B = n \rightarrow p + e^{-} + \bar{\nu}$) which becomes large when the temperature drops below a few MeV, because the neutron is heavier, by a few MeV, than the proton, it is then very disadvantaged and reaction B becomes preponderant in regard of reaction A, whereas at 1 GeV it was not noticeable.

The big fear of neutrons

Already at t = 1s there was only 1 neutron left for 3 protons and the carnage continued until 1 neutron for 7 protons at t = 200 s. In addition, a free neutron is unstable with a period of approximately 15 minutes. A huge threat for neutrons!

Let's add that the excess of 1 billionth of matter which seems arbitrary is a sensitive criterion. If the gap had been notoriously smaller, there would have been no nucleosynthesis, the even more populated photon sea would have too delayed the efficiency starting time for being productive. If the difference had been notoriously larger, the nucleosynthesis would have been too efficient and the hydrogen fuel would have been largely burnt, leaving too little for the future.

Note also the existence of the deuterium chicane, the weak link in the fusion reaction, which is a critical element of the process!

A beneficial scenario

In short, this nucleosynthesis which seems to have been aborted was in fact very fruitful, because it did exactly what had to be done for us:

- Keeping 90% of hydrogen which will be the matter of large structures and the fuel for the stars created by these structures.

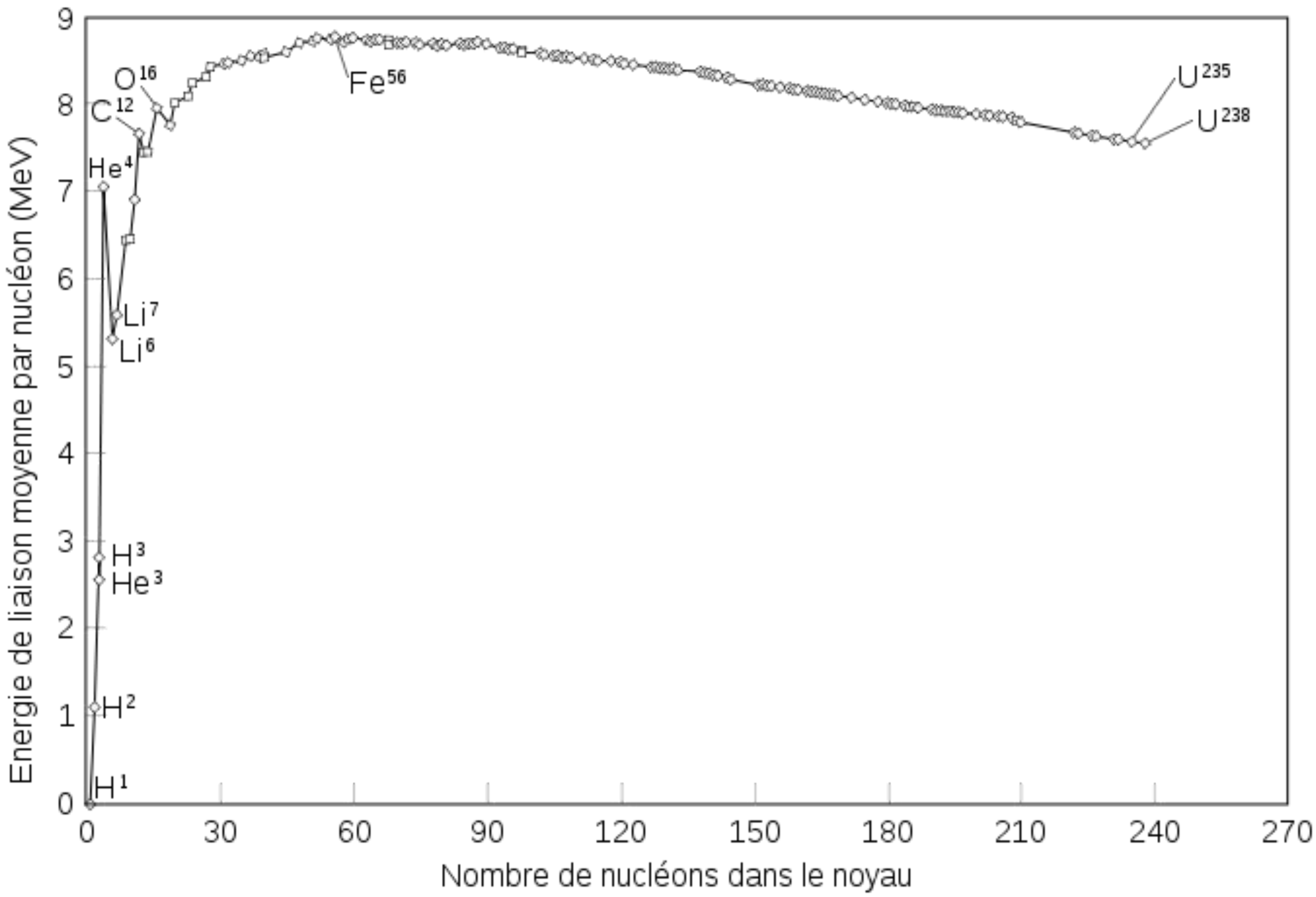
- Save neutrons by incorporating them into helium nuclei which themselves by thermonuclear fusion will synthetize elements such as carbon, oxygen, nitrogen, etc., this allowing a very complex chemistry.

This shows once again the sensitivity to the values of the parameters, a significant variation of which would have profoundly changed our destiny.

Mendeleïev periodic table

Group → / Period ↓	1	2	3		4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc		22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y		40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				**	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

Binding energy of the nucleus



In helium, the binding energy is 7 MeV per nucleon, i.e. 28 MeV for the nucleus. It is this energy that will be released during the fusion. It's about 10 million times more than a chemical combustion reaction.

Hydrostatic equilibrium

The Sun, like any star, is a ball of gas in hydrostatic equilibrium. At each point, the pressure of the gas which tends to expand it exactly compensates the gravity which, on the contrary tends to contract it.

This state of equilibrium explains the spherical shape of the Sun.

The temperature is the highest in the center of the Sun: 15.5 million degrees Kelvin. The pressure reaches 340 billion times the Earth's atmospheric pressure. The density is 158 000 kg per cubic meter.

The temperature gradually decreases as you get closer to the surface.

Hydrostatic equilibrium

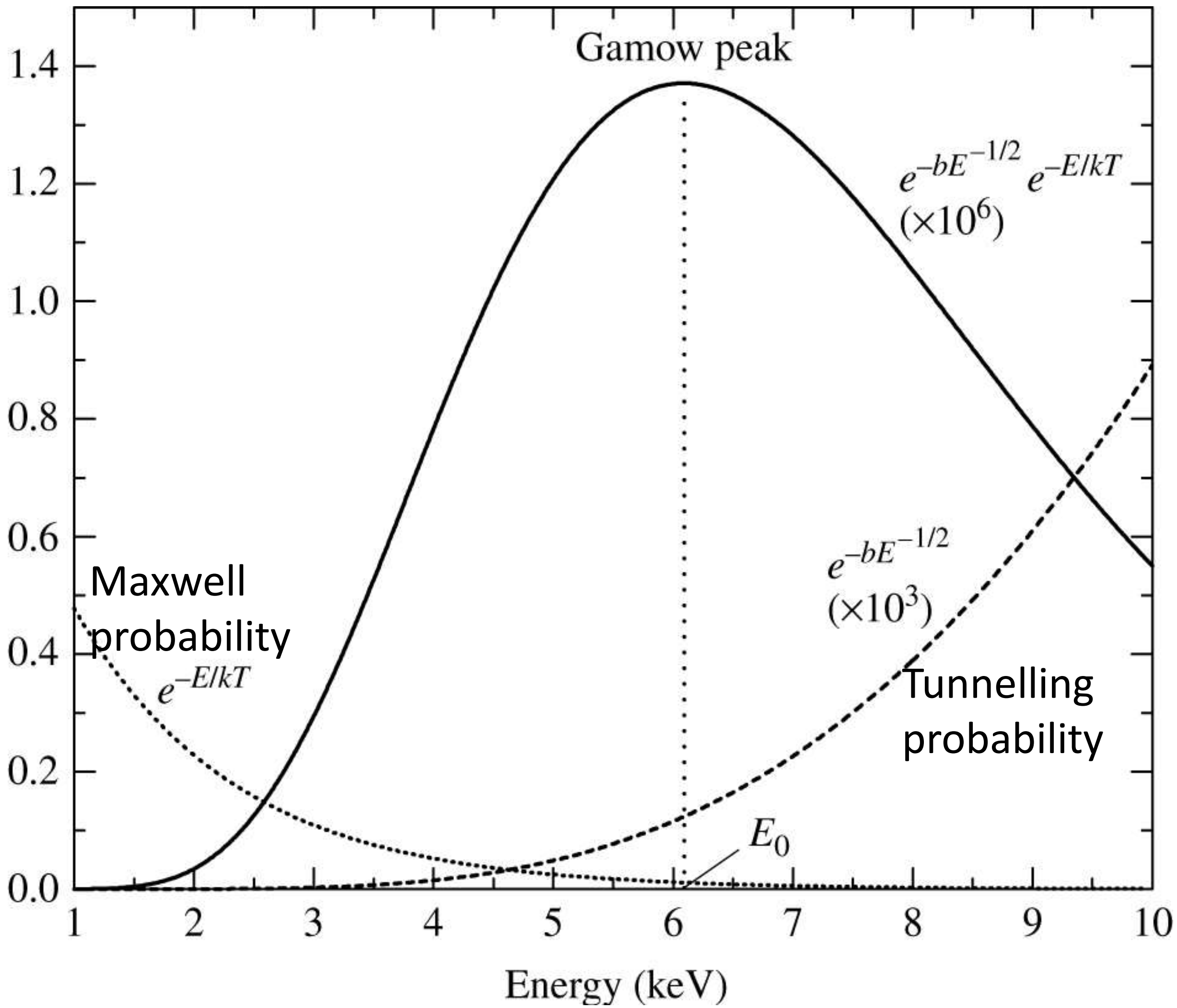
.

In the 500 km thick photosphere, where all visible light comes from, the temperature is 5800 K.

Then the temperature rises to around 100,000 K in the first layer of the, 2,500 km thick, rarefied atmosphere, of the Sun, called the chromosphere.

The temperature reaches 1 to 2 million degrees Kelvin in the upper atmosphere of the Sun, the crown, which extends over a few million km.

A difficult combustion



The energy required to fuse two protons is 1 MeV (10^{10} K). As the temperature in the center of the Sun is $15.5.10^6$K≈1.6KeV, nuclear reactions are difficult to occur. Without a tunneling effect, the reactions would not have started. The combination of the distribution of Maxwell and the tunnel effect described by Gamow shows a maximum around 6 KeV).

A difficult combustion

How with such a low temperature the Sun can shine. The answer is given by its formation and the hydrostatic equilibrium that occurred when the nuclear reactions ignited, corresponding to the state we see today.

It is its enormous mass, which compensates for the low probability of occurrence of a thermonuclear fusion reaction.

The energy per kilogram of the Sun is 0.2 mW per kg (4.10^{26}W / 2.10^{30}kg). That of the human body is around 1W per kg. We see that we shine 5000 times more than the Sun!

This illustrates the previous argument of the importance of mass. Note that the fact that it burns slowly is rather an advantage for us, because in doing so, it will still burn for a very long time !!!

Energy balance of the Sun

It is the almost exclusive supplier of energy for the Earth's surface. This huge gas ball consists mainly of hydrogen. The temperature at its center rises to 15.5 million degrees, which makes the fusion very slow.

The transformation of hydrogen into helium by nuclear fusion is accompanied by a colossal release of energy per second:

$$3.83\ 10^{26}\ \text{watts},$$

Approximately 4.10^{17} modern 1 GW nuclear reactors.
The Earth, due to the distance of its star (150 million kilometers on average) receives only 1 billionth of this energy. However, this energy is sufficient to maintain the dynamics of life and the climate.

Energy balance of the Sun

To give an idea of the phenomenal energy of the Sun, suppose that we capture its energy just for 1 second and store it in a large capacity battery.

This second of energy contains:

$$3.83\ 10^{26} \text{ joules,}$$

This energy would be enough to power the Earth for 32 years in exactly the same way as the Sun does!

Energy balance of the Sun

The formula $E = M.c^2$ shows us that the Sun transforms into energy (it becomes lighter) every second 4.26 billion kg.

It burns approximately 600 billion kg of hydrogen, because the yield is approximately 0.7% of the mass energy given by $E = Mc^2$ of hydrogen, since it is the binding energy which is released for the fusion of 4 protons in He.

The energy of high energy photons (X and γ rays) released during the fusion reactions takes a considerable time to cross the radiation and convection zones before reaching the surface of the Sun. It is estimated that the transit time of the heart to the surface is between 10,000 and 170,000 years.

Energy balance of the Sun

After passing through the convection layer and reaching the photosphere, photons escape into space, largely in the form of light. Each gamma ray produced in the center of the Sun is ultimately transformed into several million of light rays which escape into space.

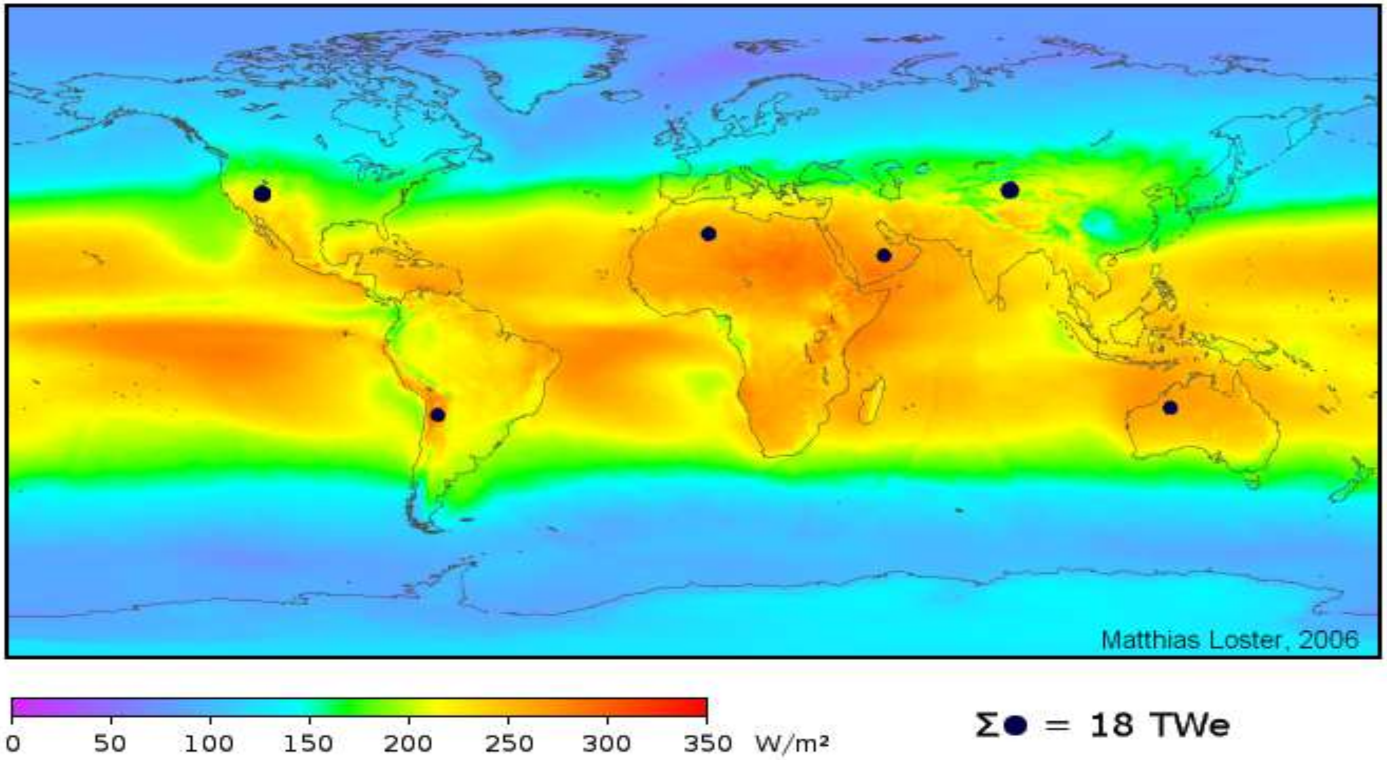
Neutrinos are also released by fusion reactions, but unlike photons they interact little with matter and are therefore released immediately. For years, the number of neutrinos produced by the Sun was measured one third lower than the theoretical value: it was the problem of solar neutrinos, which was solved in 1998 thanks to a better understanding of the neutrino oscillation phenomenon.

Energy balance of the Sun

Since its birth (approximately 4.5 billion years ago) it has transformed into helium $8.5.10^{28}$ kg of hydrogen on its 2.10^{30} kg (approximately), or approximately the mass of 10,000 Earths (Earth weighs 6.10^{24}kg)!

It therefore transformed into energy less than 0.035% of its mass (it probably lost more than that in ejection of matter) and this concerned about 6.7% of its hydrogen (Recall that its mass is composed of 75% of hydrogen and 25% helium).

Energy distribution



If we neglect the absorption by the atmosphere, each square meter at the level of the Earth's orbit, facing the sun, receives, around 1KW maximum, the day, usable by an ideal sensor (100% efficiency) perpendicular to the solar rays, and this represents only 345 W per m^2 of land on average over 24 hours.

Sun and Earth

Solar plant in California

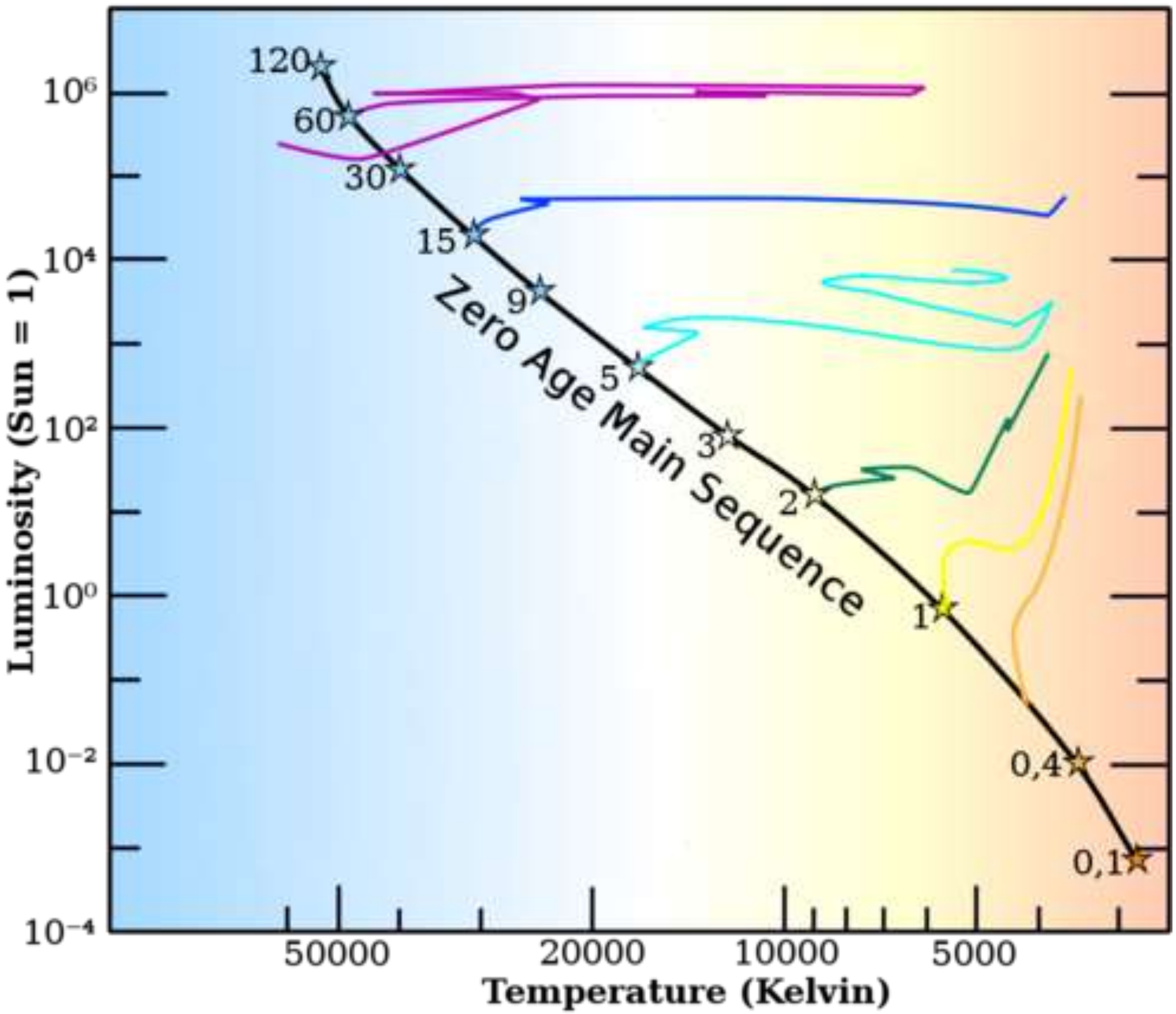
Sun, tan and life

It is pleasant to sunbathe in the Sun. But if we knew that most of the rays (photons) are at 5600 ° K, (more than 10 000K for UV) maybe that would cool some! How come we are not immediately charred?

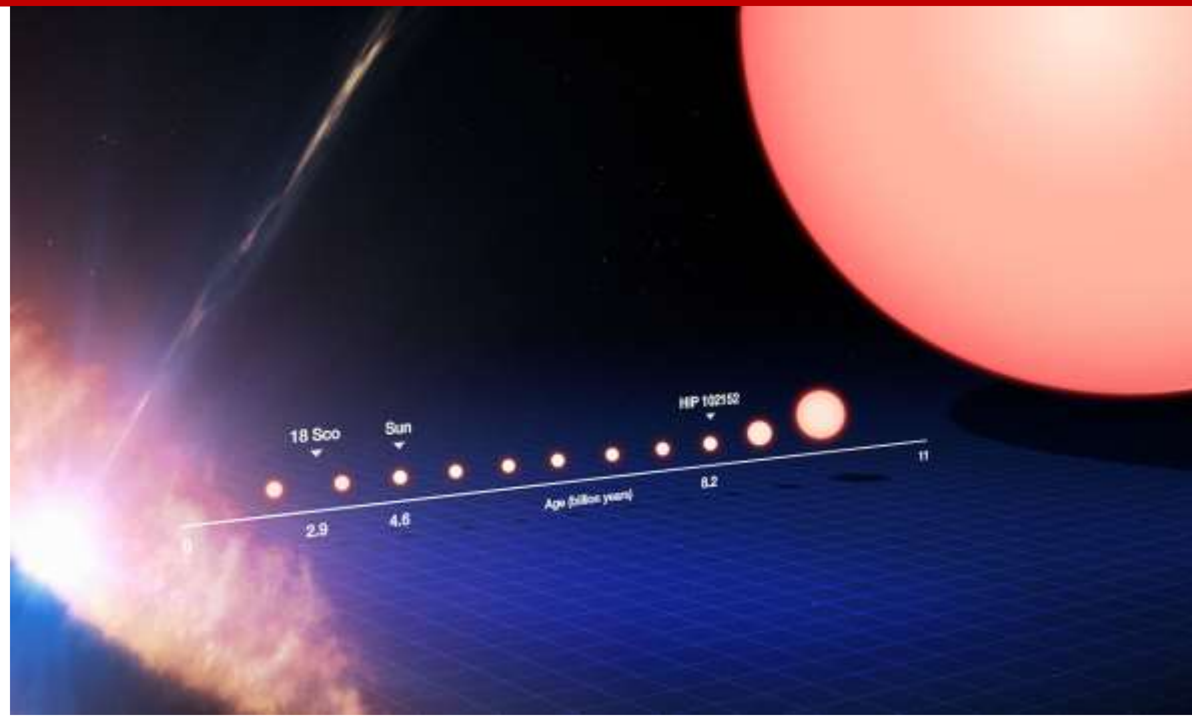
It is because, on Earth, the flow of photons is weak, the atoms of our body hit by photons, in relation to the neighboring atoms will transmit to them most of their energy which will lead to thermal equilibrium at a more reasonable temperature.

But if we concentrate these photons by a spherical concave mirror, which increases the flux, then there is to fear! This high photon temperature explains the high temperatures in the thermosphere and the terrestrial ionosphere which is a very rarefied gas where the thermal equilibrium of the gas which results from collisions between atoms is not very efficient. This also explains the high temperature of the Sun's thin gas layer where, near the Sun, even more energetic photons (UV, X) up to millions of degrees, have a flux equal to 25,000 times that received on Earth.

Star destiny

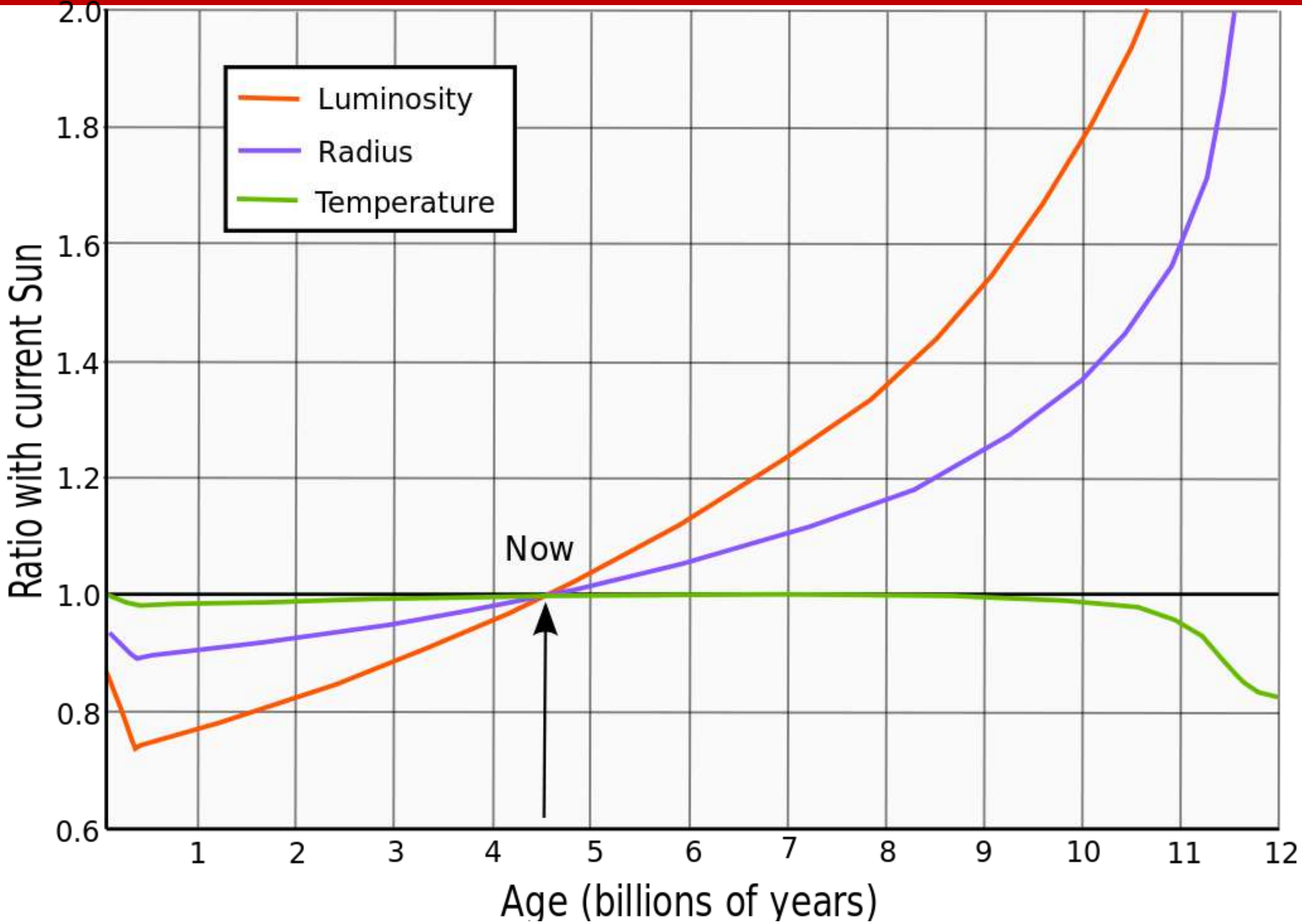


Origin and destiny of the Sun



Four and a half billion years ago a star exploded in the vicinity (in the cosmological sense: d <10 al) of the region where the Sun (our star) was going to form. It has seeded this region in various chemical elements (which we will find in the sun and the planets) and the shock wave of the explosion will cause a collapse (over hundreds of millions of years) of the gas, leading to the formation of the solar system.

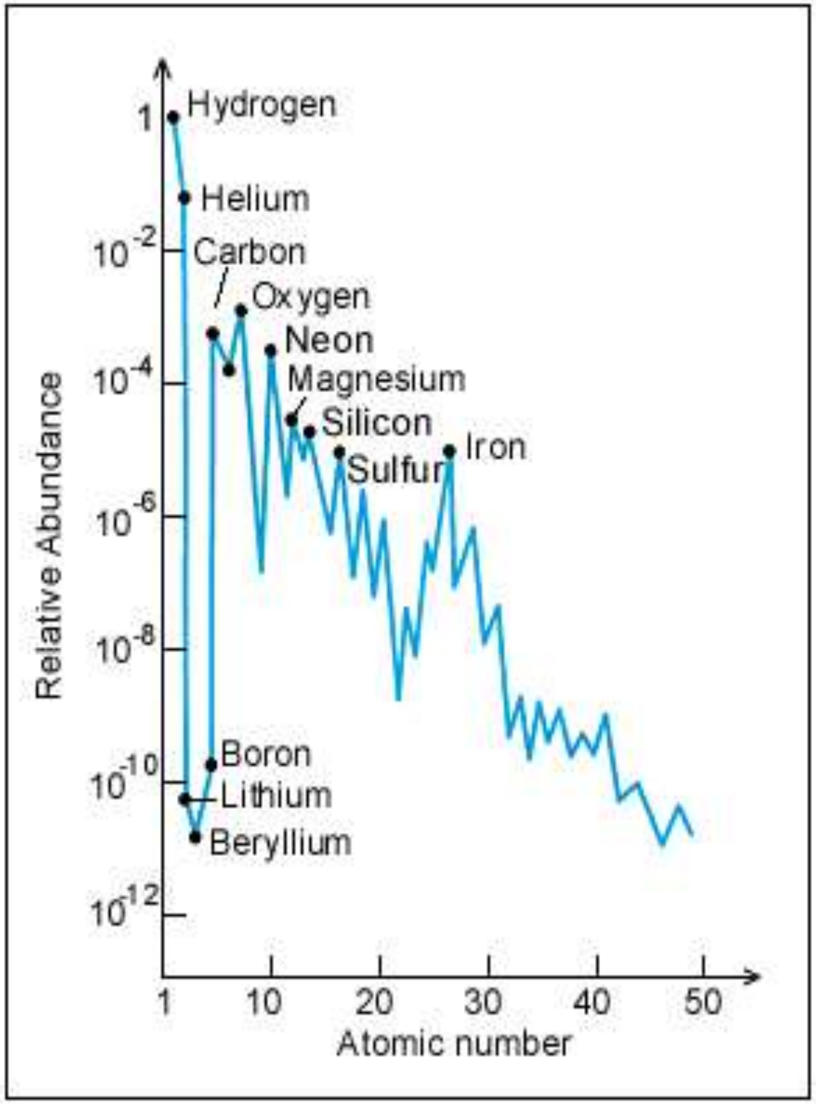
Origin and destiny of the Sun



Evolution of the solar brightness, radius and temperature, compared to the current values of the Sun. After Ribas (2010)

Chemical elements in the Sun

Abundance of chemical atoms in Sun.



Hydrogen and helium, mainly come from the primordial nucleosynthesis, even though the fusion, at work in the Sun, added a relative small part of helium.

The abundance of other elements, in the Sun, inherited from a, undoubtedly small part of the cloud, generated by the explosive fusion of the supernova looks relatively low. But per the huge mass of the Sun, this represents a mass far more greater than all the mass of these elements in all the planets of the solar system.

Chemical elements in solar system including humans

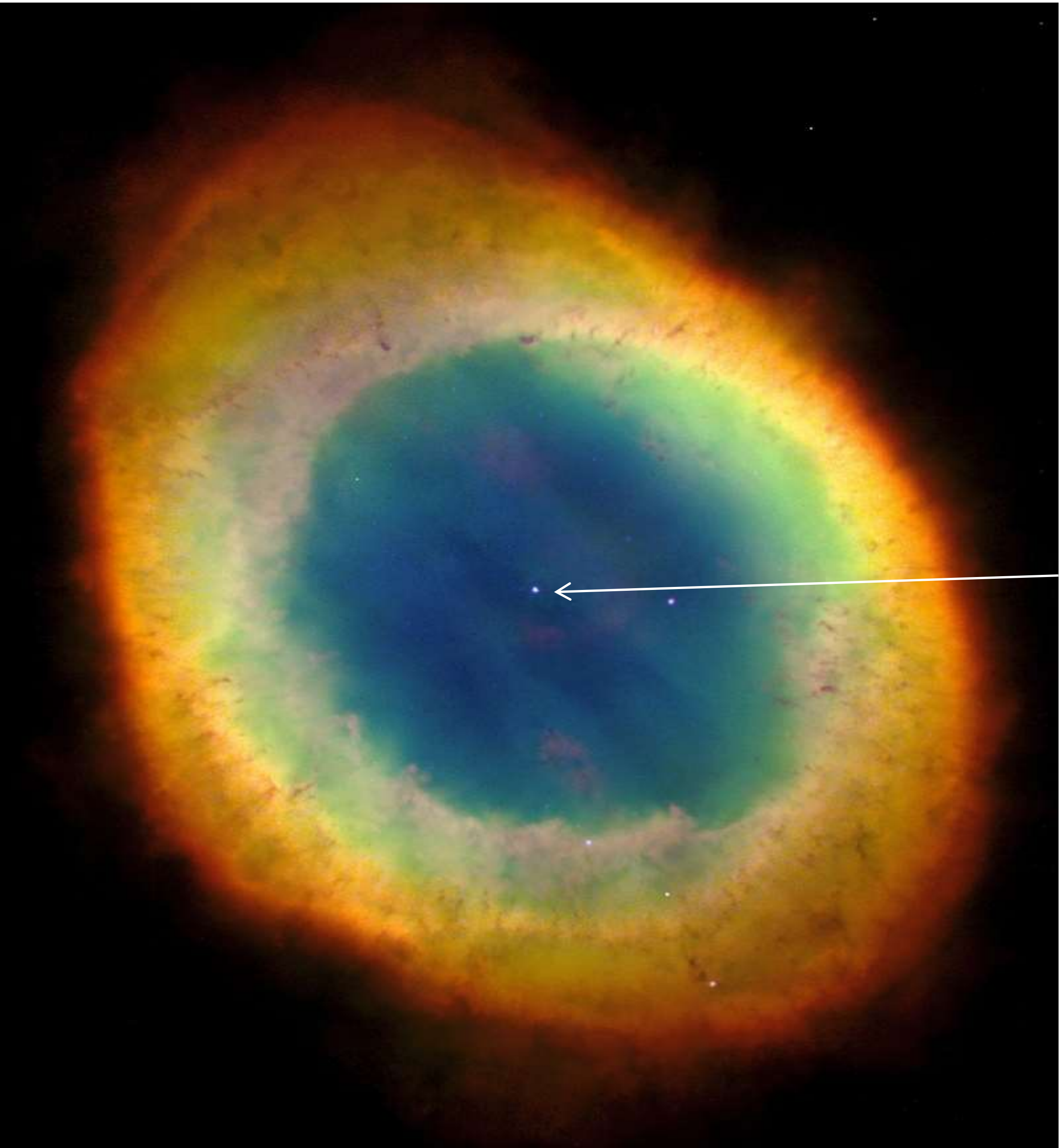
In the solar system, the abundance in chemical elements is very different. A synthesis of this abundance for the Earth, its crust, and humans, given in the next slide, shows that it is not hydrogen and helium which are dominant but rather oxygen, iron, and also carbon for human.

Chemical-atoms-synthesis

Element	Universe	Sun	Earth	Earth crust	human
Hydrogen	**75%**	**73,5%**	<1%	<1%	10%
Hélium	**25%**	**24,9%**	<0,001%	<0,001%	<0,001%
Iron		**0,16%**	**32,1%**	**5%**	0,1%
Oxygene		**0,77%**	**30,1%**	**48,6%**	**65%**
Carbon		0,029%	<1%	<1%	**18%**
Nitrogen		0,09%	<1%	<1%	**3%**
Silicium		0,07%	**15,1%**	**27,7%**	<0,1%
Calcium		<0,01%	1,5%	**3,6%**	**1,5%**
Magnesium		0,05%	**13,9%**		0,1%

Percentages in mass (red1st , orange = 2nd , green= 3th , blue 4th

Death of the Sun



At the end of its life, its envelope will expand. According to certain scenarios, it could reach Earth's orbit and encompass the Earth.

The Sun has become a white dwarf, surrounded by a nebula formed by the matter which it ejected in its final cycle.

He synthesized carbon and oxygen, but they remain trapped in the white dwarf.

So why do we talk about renewable energy when the Sun is not eternal?

The Sun was formed about 4 billion years ago and it is believed that it should still shine fairly steadily for several billion years before it begins a process in which it will evolve. After a giant and supergiant phase of red, it probably ends up as a white dwarf, the corpse of a star where nuclear reactions have ceased.

This “limited” lifespan is very large on a human scale (human history is counted in thousands or tens of thousands of years), this is why we speak of renewable energy even if it is not absolutely right.

This gives us some time to contemplate the sunset with the hope…

To see it rising, next dawn...

A-1 Einstein demonstration

Summary, with comments, in italics of this demonstration:

Einstein A. (1905) The inertia of a body depends on its energy content. Annalen der Physik, vol XVII, p 639-641. Reference [3]

Einstein will describe a thought experiment where the loss of energy from a body by radiation will be done symmetrically so that it does not affect the movement of the body. The celerity do not vary, only the mass will vary. Therefore we will associate the variation of energy (radiation type) with the variation of mass of the body.

In a one-dimensional inertial frame S_0 *(x, t)*, a body *M* is at rest. Its energy is E_0.
In a one-dimensional inertial frame S_1 *(x', t')* of relative velocity *v* with respect to S_0, this body has an energy (measured) E_1.

A-1 Einstein demonstration

The body *M* emits "symmetrically and simultaneously" two electromagnetic radiations such that the electromagnetic energy is divided into two equal parts of opposite directions.

This type of thought experience will be also used by others, Yves in the demonstration that we have proposed, for example.

The body *M* loses energy ($E0 \longrightarrow E'0$) in S_0 and likewise in S_1 where ($E1 \longrightarrow E'1$).

It remains motionless in S_0, per the symmetry of the solution and therefore its velocity does not change in S_1, it remains equal to v. In S_0, we can write:

$$\boldsymbol{E_0 = E'_0 + [L/2 + L/2] = E'_0 + L} \qquad (1)$$

A-1 Einstein demonstration

In a previous article Einstein demonstrated that:

$$L' = L\,.\gamma\,[1-(v.\cos\varphi/c)] \qquad (2)$$

With $\gamma = (1-v^2/c^2)^{-1/2}$ and where φ is the angle of propagation of the radiation with the x axis, (here ***0*** and π).

By using **(2)** for calculating *L'*, *L''* in S_1, as a function of *L* and φ, we get :

$$E_1 = E'_1 + \{ (L/2)\,.\gamma\,[(1-v/c) + (1+v/c)] = E'_1 + \gamma.\,L$$

A-1 Einstein demonstration

Einstein will consider the following energy variation, calculated from equations (*1*) and (*2*) above

$$(E_1 - E_0) - (E'_1 - E'_0) = L\,(\gamma - 1) \qquad (3)$$

In the term on the left, (E_1 - E_0) represents the kinetic energy K_0 of the body *M* (in S_1) before emission of the radiation and (E'_1 -E'_0) the kinetic energy K_1 of *M* (in S_1) after emission radiation. These energies are defined, up to an additive constant, which is eliminated in the difference ($K_0 - K_1 = \Delta W$) of the kinetic energies.

The quantity on the right therefore corresponds to the variation in kinetic energy (ΔW), linked to the emission of radiation energy

A-1 Einstein demonstration

Einstein will relate radiative energy and kinetic energy. We can rewrite the equation (*3*)

$$(E_1 - E_0) - (E'_1 - E'_0) = K_0 - K_1 = \Delta W = L\,(\gamma - 1) \qquad (4\text{-}1)$$

If $v \ll c$, then $(\gamma - 1) \approx v^2/2c^2$. In this equation we neglect terms in power of $(v^2/c^2)^n$, for $n > 1$.

In Newtonian mechanics, the kinetic energy W is : $W = \frac{1}{2} mv^2$.

In this way Einstein introduces the mass in this demonstration.

Einstein hypothesizes that we can use the form of classical mechanics of kinetic energy variation, when $v \ll c$:

$$\Delta W \sim \Delta\left(\tfrac{1}{2} mv^2\right) \qquad (4\text{-}2)$$

$$\Delta\left(\tfrac{1}{2} mv^2\right) = \tfrac{1}{2} v^2 \Delta m \qquad (4\text{-}3)$$

A-1 Einstein demonstration

The equation (*4-3*), where the celerity of ***M*** in S_1 did not vary, implies that the variation of kinetic energy can only result of the variation of the mass:

$$\tfrac{1}{2} v^2 \Delta m = \Delta W = L(\gamma - 1) : \qquad (4\text{-}4)$$

With the approximate value $(\gamma - 1 \approx \tfrac{1}{2} v^2/c^2)$, one get:

$$L(\gamma - 1) \approx \tfrac{1}{2} v^2 (L/c^2) \qquad (4\text{-}5)$$

It is this approximation, v << c, that makes the Einstein's demonstration not general, since the scope of his demonstration is limited by this condition, unlike that of Yves.
Finally, we deduce by identification and simplification:

$$\tfrac{1}{2} v^2 (\Delta m) \approx \tfrac{1}{2} v^2 (L/c^2) \rightarrow c^2 \Delta m \approx L \rightarrow \Delta m c^2 \approx \Delta W \qquad (4\text{-}6)$$

A-2 Modern demo $E = Mc^2$

From the definition of the energy in relativity,

$$E = -m.K_{\mu}dx^{\mu}/d\tau \qquad (1)$$

$\boldsymbol{K} = (\partial_t)^{\mu}$ is the Killing vector, associated to the **time** invariance, whose 4 components K^{μ} are:

$$K^{\mu} = \{1, 0, 0, 0\}$$

In the special relativity metric (SR):

$$ds^2 = -c^2d\tau^2 = -c^2dt^2 + dl^2 = \eta_{\mu\nu}dx^{\mu}dx^{\nu} \qquad \mathbf{(2)}$$

Where: $dl^2 = dx^2 + dy^2 + dz^2$,

And where: $\eta_{\mu\nu} = diag\{-c^2, 1, 1, 1\}$

Is the metric tensor of SR and $x^{\mu} = t, x, y, z$ for $\mu = 0,1,2,3$.

A-2 Modern demo E = Mc²

Let us calculate K_μ : $K_\mu = \eta_{\mu\nu} K^\mu = \{-c^2, 0, 0, 0\}$

Inserting it in *(1)* yields : $E = m.c^2 dt/d\tau \rightarrow m^2c^4(dt/d\tau)^2 = E^2$ *(3)*

Let us call ***P*** the four- momentum vector and ***p*** its space part..

The square of the norm : $P^2 = P^\mu P_\mu$

where $P^\mu = mU^\mu, P_\mu = mU_\mu$

and $U^\mu U_\mu = -c^2 (dt/d\tau)^2 + (dl/d\tau)^2 = ds^2/d\tau^2 = -c^2$

Then $\boldsymbol{P}^2 = -m^2c^2 = -m^2c^2(dt/d\tau)^2 + m^2(dl/d\tau)^2$

By using the definition of *E* in equation *(3)*, we get:

$$-m^2c^2 = -E^2/c^2 + \boldsymbol{p}^2$$

Multiplying by c^2, yields : $m^2c^4 = E^2 - \boldsymbol{p}^2 c^2$

A-3: Operator associated to a physical entity

- **Excerpt from the 2019 quantum mechanics course**

- A modern way of deriving Schrödinger's equation is to start from the Hamiltonian $H\ (x_j,\ p_j)$, which is the operator associated with the total energy of the particle (potential energy + kinetic energy).
- $$H\left(x_j, p_j\right) = E = \frac{p^2}{2m} + U(x, y, z, t)$$
- The associated wave propagation equation $\psi\ (x, y, z, t)$ is obtained by associating operators acting on the wave function, namely:
- A- The operator "multiplication noted x" by ψ for the position coordinates x_j.
- B- The operator $-i\hbar\ /\ \partial_j\psi$ for the momentum p_j.
- C- The operator $i\hbar\ /\ \partial_t\psi$ for energy E

- Note the two concepts:
- A wave function, which contains "general" information about the system,
- Operators associated with the quantities measurable by the experimenter, which will characterize human intervention, showing the interdependence between the physical world and the mind of the physicist!
- $$H\left(x_j, p_j\right) = E = \frac{p^2}{2m} + U(x, y, z, t)$$
- By performing the operations A, B, C defined above, we obtain:

$$ih\frac{\partial\psi}{\partial t} = U\psi - \frac{h^2}{2m}\Delta\psi$$

- Which is Schrödingder's equation. This process has the advantage of showing how the operators are associated with the wave function.

A-4 Demonstration of $P = \gamma m v$

General relativity is a theory of space-time. Therefore, only spacetime parameters have a physical meaning.

In Minkowski's metric in Cartesian coordinates:

$ds^2 = -c^2 d\tau^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 = dt^2(-c^2 + dx^2/dt^2 + dy^2/dt^2 + dz^2/dt^2)$

$-c^2 d\tau^2 = = dt^2(-c^2 + vx^2 + vy^2 + vz^2) = dt^2(-c^2 + v^2) \rightarrow dt^2/d\tau^2 = c^2/(c^2 - v^2) \rightarrow = \frac{dt}{d\tau} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Where *v* is the Newtonian celerity with components v_x, v_y , v_z.

The four-momentum vector (in relativity) is : ***P*** = m ***U*.**

In Cartesian coordinates, for $\mu = 0, 1, 2, 3$, we get : $P^\mu = m\, U^\mu$

$$U^\mu = \{c\frac{dt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}\} \rightarrow P^\mu = mU^\mu = m\frac{dt}{d\tau}\{c\frac{dt}{dt}, \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\} \rightarrow P^\mu = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m\{c, v_x, v_y, v_z\}.$$

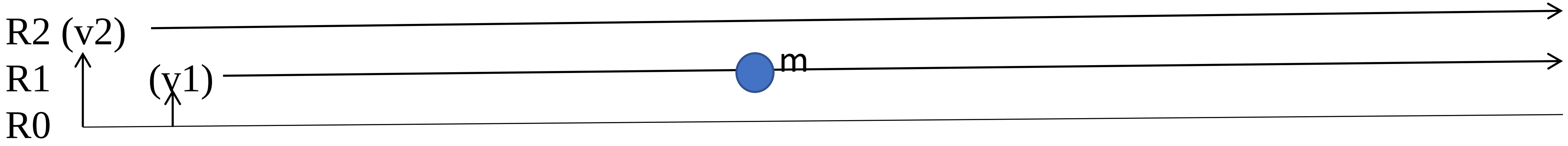
The Lorentz factor, incorporating the « temporal correction », emerges in this way in the space component of the relativistic four-momentum which corresponds to the Newtonian momentum,.

A-4 Proof of $P = \gamma mv$

A simple way to verify this formula is to use the speed composition law *u*, *v* established by Einstein in his fundamental article of 1905.

$$w = \frac{(u+v)}{1+\frac{uv}{c^2}} \qquad (1)$$

For this, we consider 3 Galilean frames R_0, R_1, R_2



R_1 has a velocity v_1 in R_0 and R_2 has a velocity v_2 in R_0. Consequently R_1 has a relative velocity $v_3 = -v_2 + v_1$ in R_2 in Newtonian mechanics.

In Newtonian mechanics, for the mass *m* in R_1, we have $P_1 = m\, v_1$ in R_0 and $P_3 = m\, v_3 = m\, (v_1 - v_2)$ in R_2. It is the same form $P_i = mv_i$ in both cases.

A-4 Proof of $P = \gamma m v$

In relativity we have to use the relativistic composition law of velocities (1) to obtain the velocity v of R_1 in R_2.

In Newtonian formalism we would have: $P_1 = m.v_1$ in R_0, and in R_2, instead of $P_3 = m.v_3 = m(v_1 - v_2)$ we get: $P_3 = mv = m\frac{v_1 - v_2}{1 - \frac{v_1 v_2}{c^2}}$

This is not the Newtonian formalism: Let's try $P = \gamma_i m.v_i$, this will give: $P_1 = \gamma_1 m.v_1$, in R_0, and in R_2: $P_3 = \gamma_v mv = \gamma_v m\frac{v_1 - v_2}{1 - \frac{v_1 v_2}{c^2}}$

Let us develop and simplify:

$$P_3 = \frac{1}{\sqrt{1 - \frac{(v_1 - v_2)^2}{c_2\left(1 - \frac{v_1 v_1}{c^2}\right)^2}}}\frac{m(v_1 - v_2)}{1 - \frac{v_1 v_2}{c^2}} = \frac{m(v_1 - v_2)}{\sqrt{1 - \frac{(v_1 - v_2)^2}{c^2}}} = \frac{m v_3}{\sqrt{1 - \frac{v_3^2}{c^2}}} = \gamma_3 m v_3$$

where $\gamma_3 = \frac{1}{\sqrt{1 - \frac{v_3^2}{c^2}}}$

This is of the form: $P_3 = \gamma_3 m.v_3$ with $v_3 = (v_1 - v_2)$, as requested.

Références

1- Bizouard C. www.annales.org/archives/x/poinca**Bizouard**.pdf (SYRTE)

2- Einstein A. (1905) Uber die grundleichungen der elektrodynamik für bewegte körper. Annalen der Physik, vol XVII, p891-921

3- Einstein A. (1905) L'inertie d'un corps dépend elle de son contenu en énergie. Annalen der Physik, vol XVII, p 639-641.

4- Albert Einstein: Œuvres choisies (traduits de l'allemand). Source du savoir – Seuil- CNRS- tome 2, p. 59-62, textes choisis et présentés par: Françoise Balibar, Olivier Darrigol, Bruno Jech et John Stachel.

5- Einstein A.(1993), « Traduction oeuvres choisies 2, Relativité I» Sources du savoir, Seuil CNRS (1993) p 60-62

6- Gourgoulhon E. (2006), « cours de Relativité générale », p 32, IAP Paris.

7- Ives H. (1952) « Derivation of the mass energy relation », J.Opt.Soc.Americ.42,8,540-543

8- Jammer (1961) Concept of mass in modern physics, Harvard University press,

9- Maxwell J.C (1891) « A Treatise on Electricity and Magnetism », Oxford,Clarendon Press,3e ed. 1891 § 792.

10- Poincaré H. (1905) « Compte Rendus de l'Académie des Sciences », 140, p1504-1508.